

Towards Addressing Surface Effects Ordinary Isotropic Peridynamic Models Position Aware Linear Solid (PALS) SAND2013-10321C

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- **Stewart Silling** – PALS concepts, technical support and guidance
- **Mike Parks** – project support



Position Aware Linear Solid (PALS)

Outline

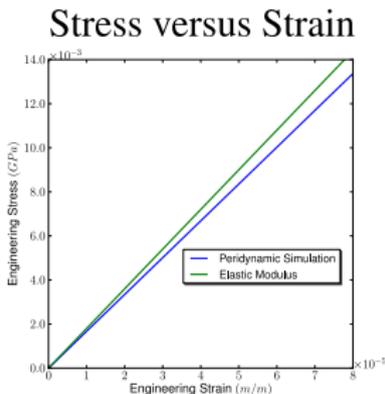
- ↪ Review the practical issue/problem of surface effects
- ↪ Introduce (PALS) & compare linear peridynamic solid (LPS)
- ↪ Selecting/creating/evaluating influence functions
- ↪ *Matching deformations*: dilatation, deviatoric
- ↪ Examples: compute dilatation, energy density for simple shear
- ↪ What do the PALS influence functions look like?
- ↪ Summary, closing comments, and path forward



What is the *Dreaded Surface Effect*?

Example: Isotropic-Ordinary Model

Axial Displacement



Stored Elastic Energy



The following related aspects contribute to the above mismatch.

- Geometric surface effects
- Nonlocal model (dilatation on surface) and model properties
- Discretization error

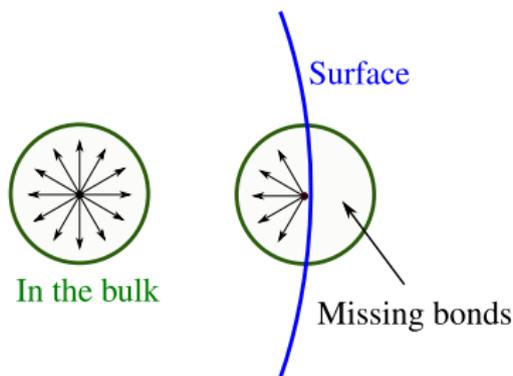


Ordinary peridynamic models

Dreaded Surface Effect

Causes relate to material points near surface

- ↪ Mathematical models assume all points are in the *bulk*
 - * Points near surface are *missing bonds*
 - * *Missing bonds* imply and induce incorrect material properties
 - * **In the bulk mathematical models are consistent**
- ↪ Isotropic ordinary materials have a *dilatation defect* at the surface

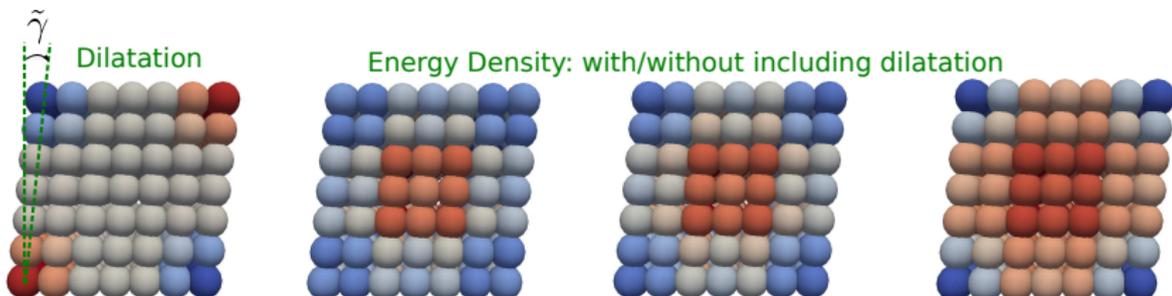


Ordinary peridynamic models

Dreaded Surface Effect

Model Problem

↪ Simple shear (more later)



Proposed Corrections

↪ *DSF*: Position aware scalar correction (March 2013)

↪ *PALS*: Today's presentation



Isotropic ordinary elastic models

Compare *LPS* with *PALS*

Kinematics

$$e = |\underline{\mathbf{Y}}| - |X| \quad \underline{\boldsymbol{\varepsilon}} = e - \frac{\theta}{3}|X| \quad \text{Bond: } \xi = x' - x = X \langle \xi \rangle$$

Linear peridynamic solid (LPS) model

$$W = \frac{1}{2}K\theta^2 + \frac{\alpha}{2}(\omega \underline{\boldsymbol{\varepsilon}}) \bullet \underline{\boldsymbol{\varepsilon}}, \quad \theta = \frac{3}{m}(\omega |X|) \bullet e$$

$$m = \omega |X| \bullet |X|, \quad \alpha = \frac{15\mu}{m}$$

PALS model

$$W = \frac{1}{2}K\theta^2 + \mu(\sigma \underline{\boldsymbol{\varepsilon}}) \bullet \underline{\boldsymbol{\varepsilon}}, \quad \theta = (\omega |X|) \bullet e$$



Selecting influence functions

Compare *LPS* with *PALS*

Linear peridynamic solid model

↪ ω is given and used for every point in mesh

PALS model

↪ ω, σ are computed for each point in mesh

↪ Initial influence functions ω_0, σ_0 given

↪ Select ω, σ as best approximations to ω_0, σ_0 subject to kinematic constraints: *matching deformations* $e^k \langle \xi \rangle = \frac{\xi \cdot H^k \xi}{|\xi|}$

$$I(\omega, \lambda^{1:6}) = \frac{1}{2}(\omega - \omega_0) \bullet (\omega - \omega_0) - \sum_{k=1}^6 \lambda^k (\omega|X| \bullet e^k - \text{Tr}(H^k))$$

$$J(\sigma, \tau^{1:6}) = \frac{1}{2}(\sigma - \sigma_0) \bullet (\sigma - \sigma_0) - \sum_{k=1}^6 \tau^k [(\sigma \varepsilon^k) \bullet \varepsilon^k - |\text{dev symm } H^k|^2]$$



Matching deformations

$$\text{Probe operator } e^k \langle \xi \rangle = \frac{\xi \cdot H^k \xi}{|\xi|}$$

Dilatation

Let probe Δ be denoted by $\Delta = XX = YY = ZZ$

$$\underbrace{\begin{bmatrix} XX & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H^1} \quad \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & YY & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H^2} \quad \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ZZ \end{bmatrix}}_{H^3}$$

Let bond ξ components be denoted by $\{a, b, c\}$

$$e^1 = \frac{\Delta a^2}{|\xi|} \quad e^2 = \frac{\Delta b^2}{|\xi|} \quad e^3 = \frac{\Delta c^2}{|\xi|}$$



Matching deformations

$$\text{Probe operator } e^k \langle \xi \rangle = \frac{\xi \cdot H^k \xi}{|\xi|}$$

Deviatoric

Let probe Δ be denoted by $\Delta = XY = XZ = YZ$

$$\underbrace{\begin{bmatrix} 0 & XY & 0 \\ XY & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H^4} \quad \underbrace{\begin{bmatrix} 0 & 0 & XZ \\ 0 & 0 & 0 \\ XZ & 0 & 0 \end{bmatrix}}_{H^5} \quad \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & YZ \\ 0 & YZ & 0 \end{bmatrix}}_{H^6}$$

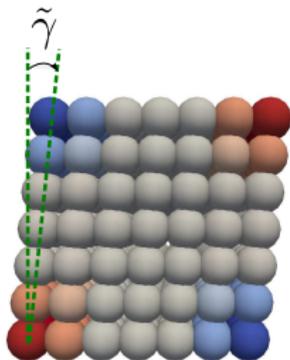
Let bond ξ components be denoted by $\{a, b, c\}$

$$e^4 = \frac{2ab\Delta}{|\xi|} \quad e^5 = \frac{2ac\Delta}{|\xi|} \quad e^6 = \frac{2bc\Delta}{|\xi|}$$



Model problem: simple shear

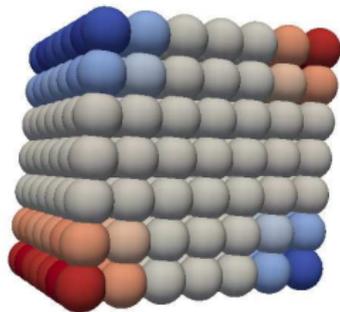
PALS versus *LPS*: expectation *dilatation* $\theta = 0$



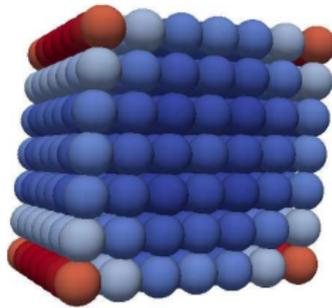
Simple shear

$$u = \tilde{\gamma}y; \quad v = 0; \quad w = 0; \quad \tilde{\gamma} = 1.0 \times 10^{-6}$$

Dilatation



LPS

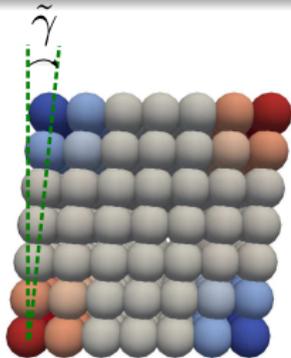


PALS



Model problem: simple shear

PALS versus *LPS*

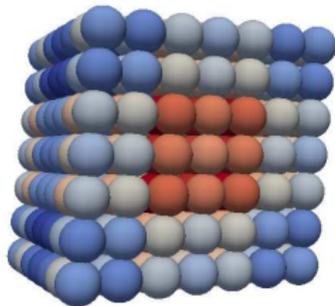


Simple shear

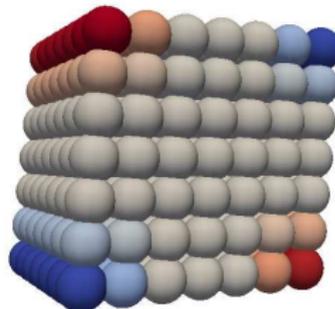
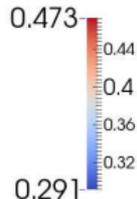
$$u = \tilde{\gamma}y; \quad v = 0; \quad w = 0; \quad \tilde{\gamma} = 1.0 \times 10^{-6}$$

$$W_L = \frac{1}{2}\mu\tilde{\gamma}^2; \quad \mu = 6.923 \times 10^{11}; \quad W_L \approx .34615$$

Stored elastic energy density



LPS

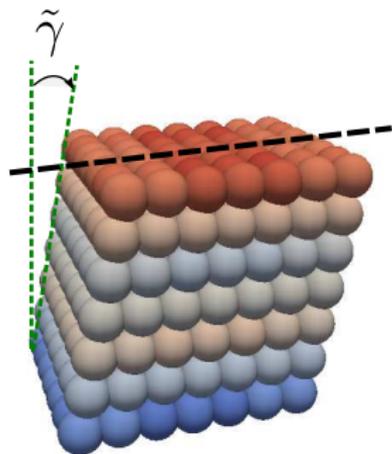


PALS



Model problem: simple shear

PALS versus LPS



Simple shear *unit cube* $h = \frac{1}{7}$

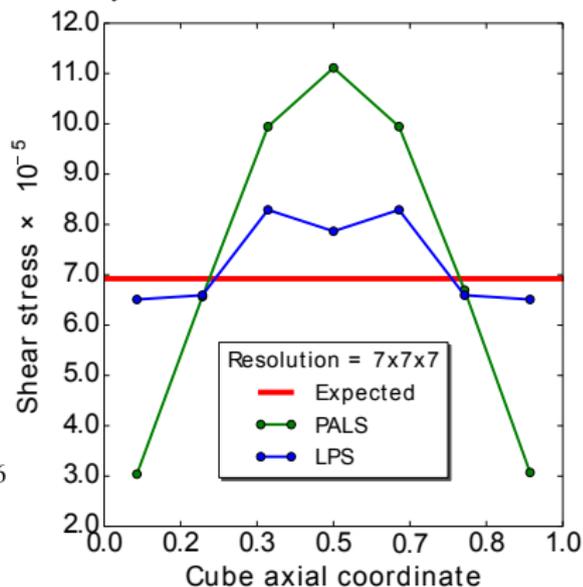
Shear stress $\sigma_{xy} = \mu \tilde{\gamma} = 6.923 \times 10^5$

$u = \tilde{\gamma}y$; $v = 0$; $w = 0$; $\tilde{\gamma} = 1.0 \times 10^{-6}$

$\mu = 6.923 \times 10^{11}$

Estimated shear stress

$\sigma_{xy} \approx \text{force density} \times h$



Model problem: simple shear bar

PALS versus *LPS*: Applied deformation

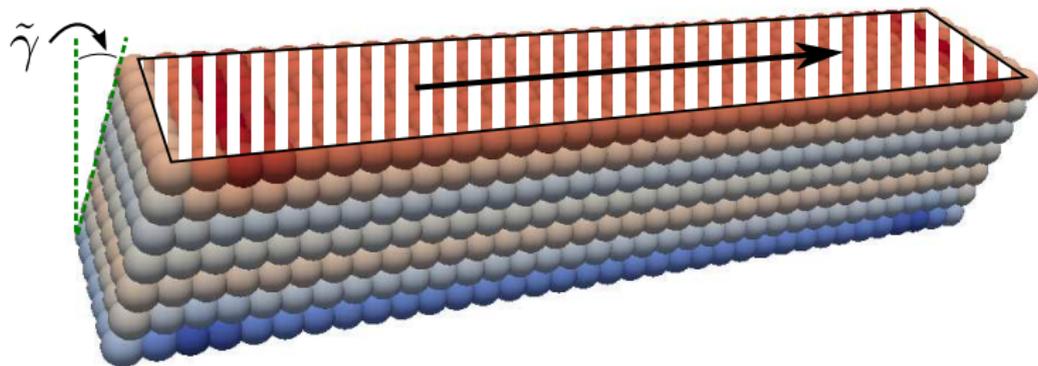
Dimension: $1 \times 1 \times 5$

Mesh resolution: $n_x \times n_y \times n_z = n_x \times n_x \times 5n_x$

Shear stress $\sigma_{xz} = \mu \tilde{\gamma} = 6.923 \times 10^5$

$u = 0$; $v = 0$; $w = \tilde{\gamma}y$; $\tilde{\gamma} = 1.0 \times 10^{-6}$

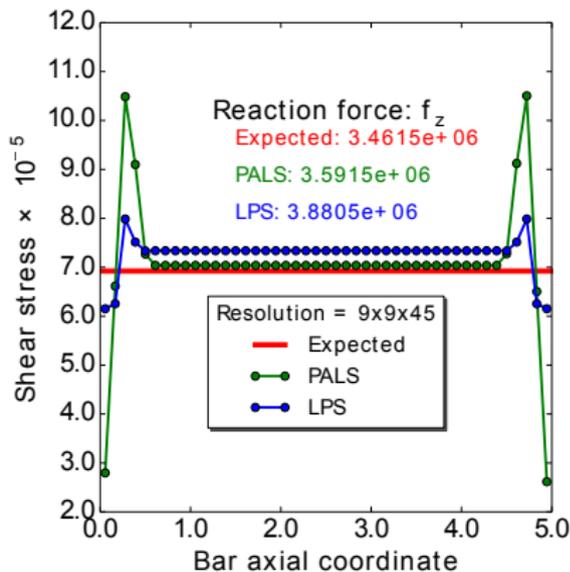
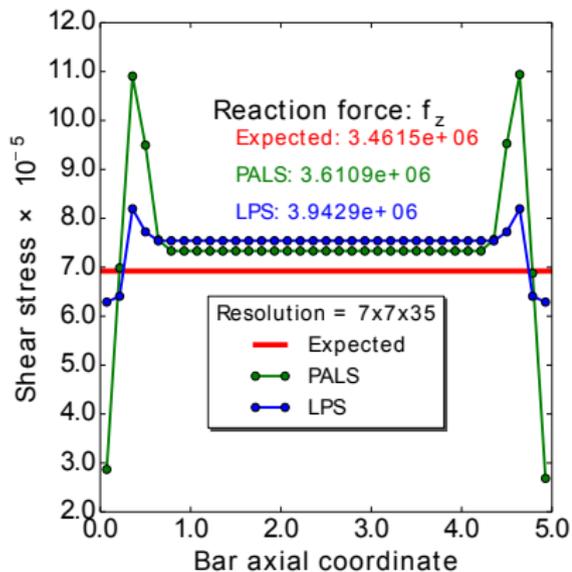
$f_z = 5 \times \sigma_{xz} \approx 3.4615 \times 10^6$



Model problem: simple shear bar

PALS versus *LPS*: Applied deformation

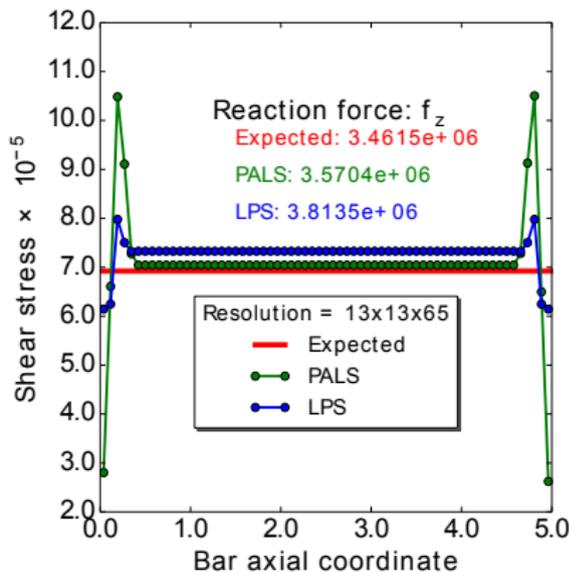
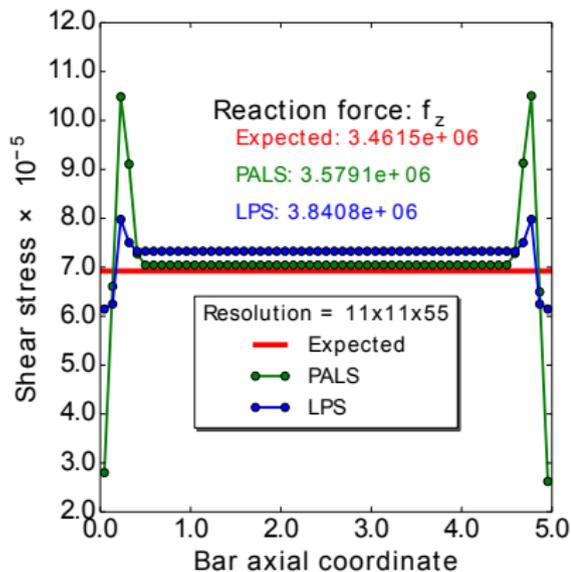
Estimated stress along top surface; Resultant force



Model problem: simple shear bar

PALS versus *LPS*: Applied deformation

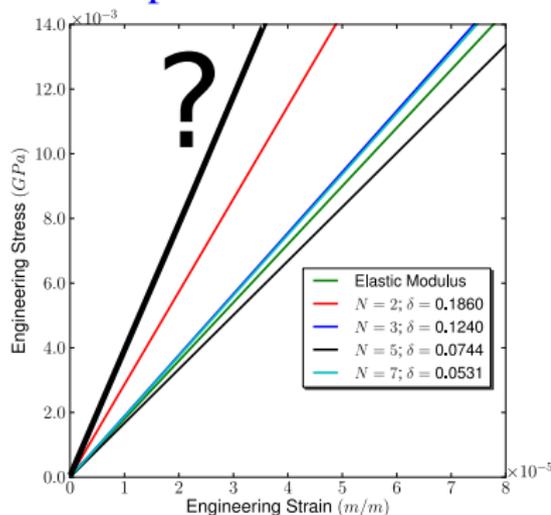
Estimated stress along top surface; Resultant force



Mesh Refinement Study

Horizon is tied to mesh element size h : $\delta = 3h$

Model problem: tensile test



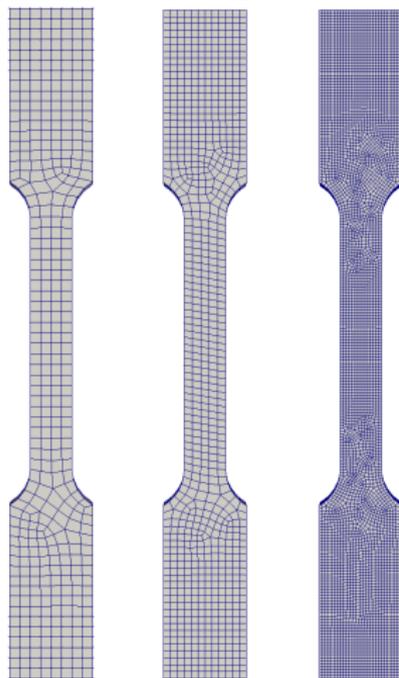
- Convergence with smaller mesh size
- Convergence slow but apparent
- Solution jumps across green line

N : number of cells thickness

$N = 2$

$N = 3$

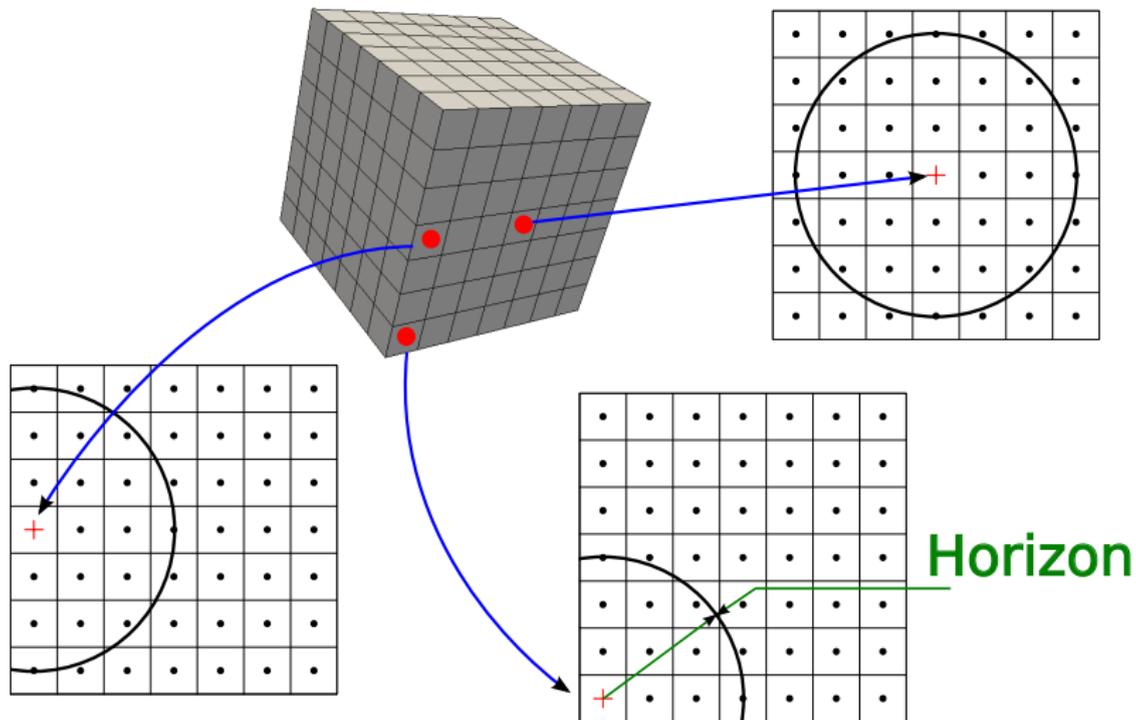
$N = 7$



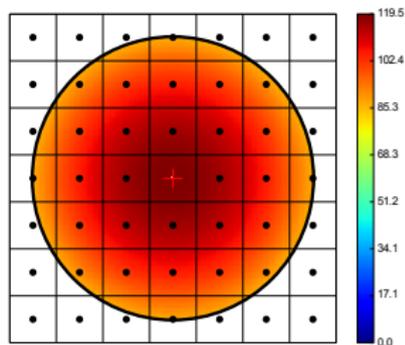
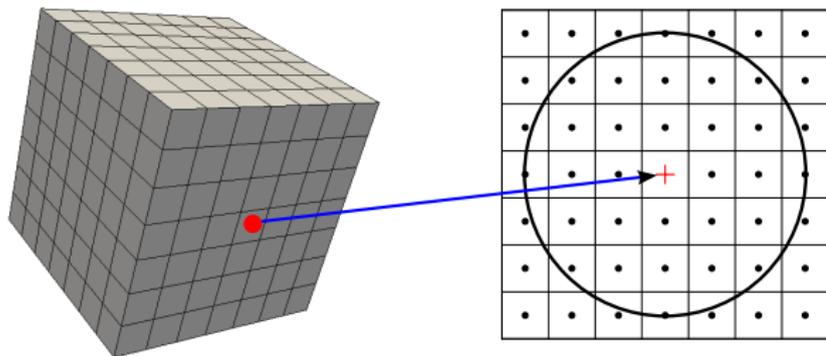
inches



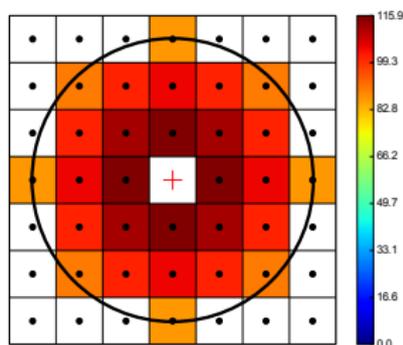
What do the *PALS* influence functions look like?



PALS dilatation influence function: FACE



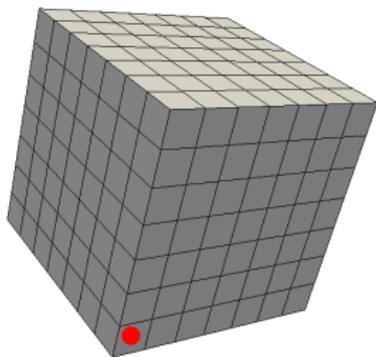
Smooth



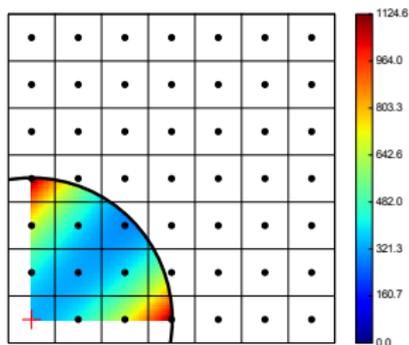
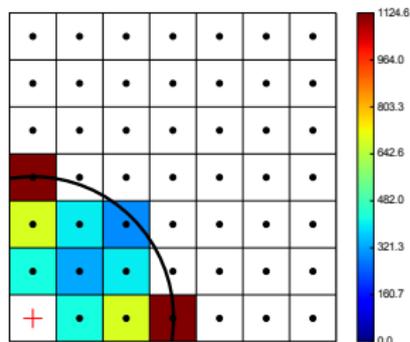
Mesh



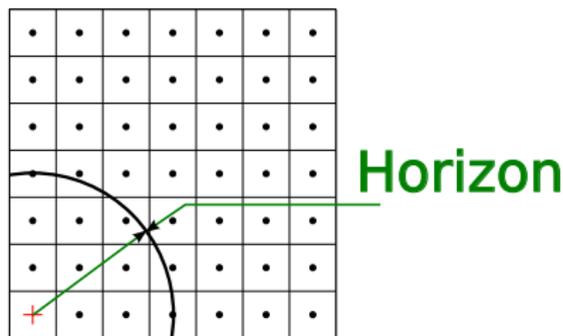
PALS dilatation influence function



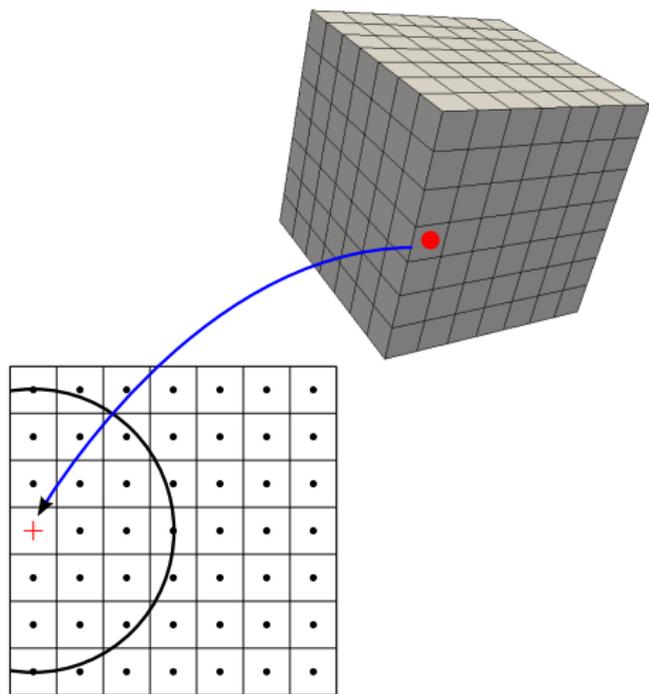
Mesh



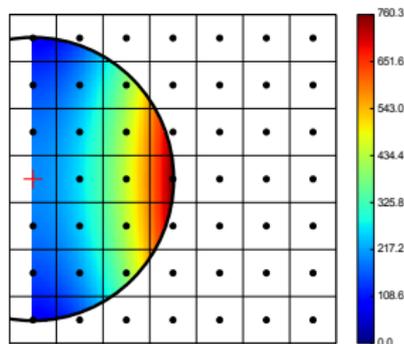
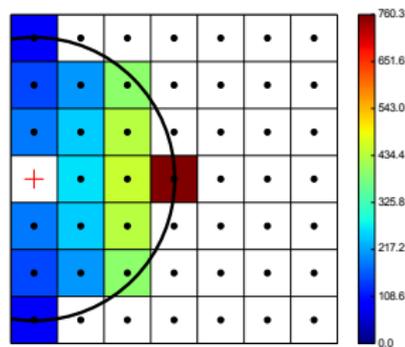
Smooth



PALS dilatation influence function



Mesh



Smooth



Position Aware Linear Solid (PALS)

Summary

- ↪ Reviewed the practical issue/problem of surface effects
- ↪ Introduced novel *Position Aware Linear Solid* model (PALS)
- ↪ PALS dilatation and energy density correct in pure shear
- ↪ Using PALS *may* require some form of stabilization
- ↪ PALS forces *may* have some non-physical forms; zig/zag



Position Aware Linear Solid (PALS)

Closing Comments and Path Forward

Comments

- ↪ Surface effects remain an important problem to be addressed
- ↪ Some indication that the appropriate scaling of moduli at the surface depends upon the displacement field (even for small displacements)

Path Forward

- ↪ *Augmented PALS* (APALS) model
 - * Augmented functional which includes energy density
 - * Couples Lagrange multipliers with displacements
 - * May address both stability and non-physical forces (*zig-zag*)
 - * Additional and unwanted complexity

