Limited Artificial Viscosity and Hyperviscosity Based on a Mollified Nonlinear Hybridization Method

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The classical artificial viscosity method [1] often suffers from too much numerical viscosity away from the shock where the method is absolutely necessary. The viscosity used to capture the shock is applied to flow structures that are not shocked resulting in needless error. A common approach to defeat this issue is to modify the viscosity with a "limiter"[2]. The construction of such limiters is a cottage industry of sorts and a number of approaches can be used. We elucidate some of the techniques we have employed to this end. A secondary impact of the approach is the use of more optimal coefficients for the viscosity itself. The coefficients can be derived through analysis of the Rankine-Hugoniot relations [3]. We repeat these analyses, but include the relative positions of the variables in the analysis producing a modification of the standard analysis results. We provide results that lend credence to the quality of our results.

The limiter is defined by the nonlinear hybridization technique developed in [4]. A function is defined as the normalized ratio of second-to-first derivatives, or a function of this ratio. The original method was used to define a method that merged low-order monotonic methods with high-order (non-monotonic) methods to produce non-oscillatory results near shocks (discontinuities), and high-order results away from them. The standard form is the following as applied to a flux,

$$f_{j+1/2} = \phi_{j+1/2} f^{\text{low}}_{j+1/2} + \left(1-\phi_{j+1/2}\right) f^{\text{high}}_{j+1/2}, \tag{1}$$

where ϕ is the limiter and the update is applied in conservation form. In the case of artificial viscosity, the limiter is applied to allow the viscosity to be modified and the usual "Q" takes the place of the low-order monotonic method,

$$\tilde{Q}_j = \phi_j Q_j, \tag{2}$$

and the high-order method is the integration method without any viscosity at all. Earlier in the effort we examined the use of flux-corrected transport [5] with similar success, but the nonlinear hybridization was deemed more flexible and extensible to unstructured meshes. Moreover, the properties of the limiter could be made more mesh independent and abiding to important symmetry and invariance characteristics.

The limiter allows the use of analytically derived properties. The analysis generally proceeds from the Rankine-Hugoniot relations,

$$\left[p_1 - p_0\right] = W_s\left[u_1 - u_0\right] \rightarrow p_s = p_0 + W_s\left(u_1 - u_0\right); W_s = \rho_0\left(c_0 + s_0\,\Delta u\right). \tag{3}$$

This analysis does not take the position of the variables into account. In a staggered mesh, the pressure and velocity are not in the same positions. We can consider that each variable is piecewise constant for the purposes of analysis. This gives us to separate formulations to consider, for either the element,

$$p_s - p_j = W_{S,j}\left(u_s - u_{j-1/2}\right); p_s - p_j = W_{S,j}\left(u_{j+1/2} - u_s\right), \tag{4}$$

or the edge. This element is what we want because the artificial viscosity is found by solving the equations for the shocked pressure,

$$p_s = p_j + \tfrac{1}{2} W_s\left(u_{j+1/2} - u_{j-1./2}\right). \tag{5}$$

This analysis can then be carried forward to define analytic linear and quadratic viscosity coefficients linear and quadratic,

$$c_1 = \tfrac{1}{2}\rho_j c_j; c_2 = \tfrac{1}{4}\rho_j s_j\,. \tag{6}$$

At this point we can refine these ideas with two enhancements: the limiter can be mollified to allow better smoothness and general mathematical properties, and the use of hyperviscous diffusion. The limiter can be replaced by a smoother function, $g(\phi)$,with several specific properties, i.e., $g(0)=0, g(1)=1$ and vanishing derivatives with respect to ϕ at zero. Power functions such as ϕ^2 and Gaussians or hyperbolic tangents can meet these criteria. The hyperviscosity can help to more effectively control small-scale oscillation that invariably pollutes solutions. The hyperviscosity can be defined by applying a symmetric filter (average) to the viscosity,

$$\tilde{Q}_{j,4} = \overline{Q}_j - Q_j\,, \tag{7}$$

this operation can be applied recursively to produce higher order viscosities. This viscosity can be combined with the original limiter to produce a final form,

$$\widehat{Q}_j = \phi_j \widetilde{Q}_j + \left(1-\phi_j\right)\tilde{Q}_{j,4}\,. \tag{8}$$

The combination of the limiter with the hyperviscosity produces sharp shock transitions while effectively reducing the amount of high frequency noise emitted by the shock. Unfortunately, it is somewhat less effective with stronger shocks. These characteristics will be demonstrated computationally.

References

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