A Multiscale and Multifidelity Framework for Simulating Flow Control Systems

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy’s National Nuclear Security Administration under contract DE-AC04-94AL85000.
Sandia and Fluid Flows

Aerodynamics

MHD

Combustion

Homeland Security

Shock Hydrodynamics

Premo (Ober et al.)

Allegra HEDP (Robinson et al.)

MPSalsa (Shadid et al.)

Fuego (Domino et al.)

Allegra (Robinson et al.)
Application of optimization to transient, *compressible* flows is largely untapped...

Transient optimization and control problems are increasingly important:

- Steady-state solutions do not capture critical physics:
  - aeroacoustics, combustion instabilities, shock/BL interaction, wakes, ...
- Next generation systems will likely use *active* design/control techniques.

Algorithmic Challenges:

- Complex geometries
- Unsteady flow physics
- Localized, broadband physics
- Gradient evaluation
- Storage of time-history
- Complex problem setup
- Large-scale space-time problems
Optimization of Unsteady Compressible Flows

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- DG + Optimization = SAGE
  - Unstructured meshes.
  - High accuracy, low-dissipation.
  - Multiscale / zonal models.
  - Adjoint methods.
  - Efficient I/O, checkpointing, TDD
  - Object-oriented software design.
  - Parallel algorithms.
Discontinuous Galerkin

Key Points:

- A hybrid between finite element and finite volume methods.
- Solutions continuous in elements, discontinuous across element interfaces.
- Elements are coupled via numerical fluxes on element interfaces.

Advantages:

- Spectral accuracy on arbitrary meshes,
- Local $hp$-refinement,
- Diagonal mass matrix,
- Weak boundary conditions,
- Local conservation,
- Low communication overhead.

Disadvantages:

- More unknowns for same accuracy,
- Potentially high FLOP count,
- Aliasing at high-orders,
- Requires high-fidelity geometry,
- Limiters required for shocks ($p > 0$),
- Must exploit inherent flexibility.

Bottom-line:

High accuracy, high flexibility make DG particularly promising for transient, multi(scale, model, fidelity) optimization problems, but there is no free-lunch...
Status of SAGE Flow Solver

- Arbitrarily high-order discontinuous Galerkin spatial discretization,
- Explicit and implicit time-advancement (checkpointing on the way)
- Designed to support transient optimization problems,
- Supports **multifidelity** and **multiscale** models,
- ℓVMS approach for element-by-element subgrid-scale modeling,
- Already validated for a variety of laminar and turbulent flows . . .

with Srinivas Ramakrishnan (Rice)

with Matt Barone (Sandia)
DG + Optimization: SAGE

- Continuous (and/or discrete) adjoint formulation in space:
  - Adjoint PDEs are discretized using DG, similar to State equations.
  - Allows for accurate, stable, discretizations of both state and adjoint.
  - Enables different resolutions to be used for state and adjoint.
  - Obviates difficulties with non differentiable numerical fluxes and limiters common in DG (and FV) discretizations.
  - Provides insight into the physics of sensitivity systems and boundary conditions.
  - With particular choice of adjoint flux, can also be discrete adjoint at same resolution!

- Discrete adjoint in time: Runge-Kutta, Backward Euler, Midpoint

- Supports: Advection-Diffusion, Burgers, Wave, Euler, and Navier-Stokes

- Can utilize Sandia’s DAKOTA and MOOCHO optimization tools...

- Allows for time-domain decomposition techniques
  (Bartlet, Collis, Heinkenschloss, van Bloemen Waanders, 2004).

- Generic solver/optimization interface mimics mathematical formulation and optimization problem setup...
Modeling Systems of Sensors and Actuators

- Supports multiple Obs and Ctrl objects

\[
J(y, u) = \sum_{n=1}^{N_{\text{Obs}}} J_{\text{Obs}_n}(y) + \sum_{m=1}^{N_{\text{Ctrl}}} J_{\text{Ctrl}_m}(u)
\]

- Each Obs and Ctrl are *self-contained* and interact with the State and Adjoint as in continuous adjoint formulation.

- Design makes adding new Obs and Ctrl straightforward.
SAGE Implementation

- C++ with STL and all kernel computations using ATLAS/BLAS.
- Supports: Advection-Diffusion, Burgers, Wave, Euler, Navier-Stokes
- Designed from the ground up for parallel execution using MPI-2 and MPI-IO.
- Excellent scaling for flow solver, even on small 2-d problems ($\approx 10,000$ elements).
- Scaling of optimization problems same as flow solver, so far... (Note that optimization problem is small — only 576 elements!)

- Runs on: Linux, Mac OS-X, Cygwin, Alpha, SGI, most Sandia/DOE platforms.

Parallel speedup of DGM Solver.

Parallel speedup of DGM-Opt.
Sandia Enabling Technologies

SAGE leverages Sandia’s Object Oriented Toolkits

- **Trilinos (Heroux, et al.)**
  - Distributed, parallel sparse vectors and matrices
  - Time discretization
  - Spatial discretization (coming soon)
  - Nonlinear solvers (Newton, quasi-Newton, Picard, …)
  - Linear solvers (Parallel direct, Krylov, …)
  - Eigensolvers (parallel, iterative)

- **Dakota (Eldred, et al.)**
  - Gradient based optimization: NCG, SQP, BFGS, …
  - Genetic algorithms, direct search
  - Surogate-based optimitzation
  - Multifidelity optimization
  - Uncertainty quantification

- **Zoltan (Devine, et al.)**
  - Graph and hyper-graph parallel partitioners
  - Dynamic load-balancing
  - Data migration and redistribution
Acoustic Scattering from a Circular Cylinder

- Conditions: zero mean flow, incident planar acoustic wave, $\lambda/d = 2.5$.
- Discretization: 6832 quadrilaterals with $p = 6$, sponge layer on farfield.
- Models: Euler equations near the cylinder, wave equation in the farfield

- Excellent agreement with theory
- Note clean density near solid boundaries — this is *very* hard to achieve with typical high-order aeroacoustics codes...
Cylinder-Vortex Interaction

- Conditions: inviscid, \( M_\infty = 0.3 \) mean flow.
- Vortex: Location \((x_0, y_0) = (-9.0, 0.25)\), core radius \( R_c = 0.4 \), maximum velocity \( v_{\theta\text{max}} = 0.5 \).
- Discretization: 2224 quadrilaterals, hybrid polynomial order \( p = 5 \) in the vortex path, \( p = 3 \) elsewhere.

Mesh and vortex.  Close up of mesh and vortex.  Pressure
Cylinder-Vortex Interaction

(Euler + Wave Equation)

- Conditions: inviscid, $M_{\infty} = 0.3$ mean flow.
- Vortex: location $(x_0, y_0) = (-9.0, 0.25)$, core radius $R_c = 0.4$, maximum velocity $v_{\theta \text{ max}} = 0.5$. 

![Cylinder-Vortex Interaction Diagram](image)
Cylinder Wake Control

- Conditions: $Re = 100$, $M_\infty = 0.5$.  

- DG using $N_e = 576$ quadrilaterals with $p = 4$; RK4 in time: $N_t = 2,000$, $\Delta t = 0.0015$.  

- Consider both unsteady and steady suction/blowing.  

- Full State tracking objective.

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![Element mesh](image)

Uncontrolled $\rho u$ at $Re = 100$.  

Uncontrolled $\rho u$ at $Re = 20$
Cylinder Shedding: Streamwise Momentum $\rho u$

No Control

$t = 0.2$

$t = 0.8$

$t = 1.4$

$t = 2.0$
Cylinder Shedding: Streamwise Momentum $\rho u$

Optimal Steady Control

- $t = 2.0$
- $t = 14.0$
- $t = 30.0$
- $t = 40.0$
• BVI occurs in low speed descending flight conditions. results in high amplitude impulsive noise that radiates towards the ground.

• Can optimal control lead to new techniques that alleviate BVI noise?

• Optimal control of unsteady compressible flows is very challenging…
Min \( J(y, g) \)
such that
\[
\mathcal{N}(U(Y), g) = 0 \quad \text{in } \Omega_{\text{near}}
\]
\[
\mathcal{F}(\overline{Y}, \gamma) = 0 \quad \text{in } \Omega_{\text{far}}
\]

- \( \mathcal{N}(U(Y), g) = 0 \) near-field equations
- \( \mathcal{F}(\overline{Y}, \gamma) = 0 \) far-field equations
- \( U \) near-field flow variables
- \( y \) far-field fluctuation variables
- \( g \) on-blade control
Multimodel Simulation for Optimal Control of BVI
(Navier–Stokes & Linearized Euler Equation)

- Conditions: viscous, $M_\infty = 0.3$ mean flow.
- Vortex: location $(x_0, y_0) = (-6.0, 0.25)$, core radius $R_c = 0.15$, maximum velocity $v_{\theta_{\text{max}}} = 0.5$.

Problem setup for BVI
Scattered Pressure Contours

No Control

$t = 1.76$

Optimal Control

$t = 1.76$

$t = 2.16$

$t = 2.56$

$t = 2.96$
Scattered Pressure Time Histories

Station 1

Station 2

Station 3

Station 4
Adjoint Variable $\lambda_4$)

$t = 2.56$

$t = 2.16$

$t = 1.76$

$t = 1.40$
Change in Vorticity

\begin{align*}
t &= 1.76 \\
t &= 2.16 \\
t &= 2.56 \\
t &= 2.96
\end{align*}
• Drag is slightly increased in the first half of the optimization time window.

• Peak lift is noticeably reduced.

• Lift gradient is reduced $\rightarrow$ reduced noise generation.
Closing Comments

- **SAGE** framework is operational for transient, multimodel, multiscale problems with optimization capability.
- Applied **SAGE** to several problems:
  - Acoustic scattering from a circular cylinder
  - Cylinder-vortex interaction
  - Control of vortex shedding
  - Bell AH-1 rotor vortex interaction
- Formulated and implemented multimodel optimization capability (i.e. near-field/far-field coupled adjoint methods)
- Successfully applied multimodel optimal control for BVI model problem: 12db noise reduction.
- **SAGE** is ready for use in simulation and optimization of other (closed loop) flow control systems...
- Sandia provides extensive *enabling technologies* for a wide range of simulation and modeling!
Discontinuous Galerkin Method

Strong form:
\[ U_t + F_{i,i} - F_{i,v}^v = S, \quad \text{in } \Omega \]
\[ U(x, 0) = U_0(x), \quad \text{at } t = 0 \]

and appropriate boundary conditions on \( \partial \Omega \).

Partition \( \Omega \) into \( N \) subdomains \( \Omega_e \).

\[
\int_{\Omega_e} \left( W^T U_t + W^T_i (F_{i,v} - F_i) \right) \, dx + \int_{\partial \Omega_e} W^T (F_n - F_{n,v}) \, ds = \int_{\Omega_e} W^T S \, ds
\]

Introduce numerical fluxes \( F_n(U) \rightarrow \hat{F}_n(U^-, U^+) \) and sum over all elements

\[
N \sum_{e=1}^N \int_{\Omega_e} \left( W^T U_t + W^T_i (F_{i,v} - F_i) \right) \, dx + \sum_{e=1}^N \int_{\partial \Omega_e} W^T \left( \hat{F}_n(U^-, U^+) - \hat{F}_{n,v}(U^-, U^+) \right) \, ds = \sum_{e=1}^N \int_{\Omega_e} W^T S \, ds \quad \forall W \in \mathcal{V}
\]

Benefits: High accuracy, unstructured, local \( hp \)-refinement, local conservation, . . .

Note: boundary conditions (and controls) set weakly through numerical fluxes . . .
Numerical Inviscid Flux

\( \hat{F}_n(U^-, U^+) \approx F_n(U) \)

Monotone property of numerical inviscid flux:

- Consistent with true flux: \( \hat{F}_n(U, U) = F_n(U) \)
- A nondecreasing function of \( U^- \)
- A nonincreasing function of \( U^+ \)

Specific Choices:

- Lax-Friedrichs, Steger-Warming, vanLeer, Roe, etc…
  (see e.g., Toro 1999 and Cockburn 1999)
- Example: \textit{Lax-Friedrichs flux}

\[
\hat{F}_n(U^-, U^+) = \frac{1}{2} \left[ F_n(U^-) + F_n(U^+) + \lambda_m(U^- - U^+) \right]
\]

where \( \lambda_m \) is the maximum, in absolute value, of the eigenvalues of the Flux Jacobian \( A_n = \partial F_n / \partial U \)
**Numerical Viscous Flux**

\[ \hat{F}_n^v(U^-, \sigma^-, U^+, \sigma^+) \approx F_n^v(U, \nabla U) \]

- Bassi-Rebay, Baumann-Oden, local-DG, interior-penalty …
  (see e.g., Arnold et al. 2001 and Cockburn 1999)

- Example: *Bassi-Rebay flux*

  First compute a “jump savvy” gradient of the state, \( \sigma_j \sim U_j \) by solving for

  \[
  \sum_{e=1}^{N} \int_{\Omega_e} W^T \sigma_j \, dx = - \sum_{e=1}^{N} \int_{\Omega_e} W \cdot j^T U \, dx + ds + \sum_{e=1}^{N} \int_{\partial \Omega_e} W^T \hat{U}(U^-, U^+) n_j \, ds
  \]

  \( \forall W \in \mathcal{V} \) and \( j = 1, 2, 3 \), where

  \[ \hat{U}(U^-, U^+) = \frac{1}{2} (U^- + U^+) \]

  The Bassi-Rebay numerical flux is then computed using

  \[ \hat{F}_n^v(U^-, \sigma^-, U^+, \sigma^+) = \frac{1}{2} (F_n^v(U^-, \sigma^-) + F_n^v(U^+, \sigma^+)) \]