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Contents

SIAM OP17

Every three years, the SIAM Optimization conference provides an excellent opportunity to survey the developments in our field as well as to catch up with and make new colleagues. This is one setting where taking a shortest path between conference rooms is rarely optimal! In this issue we highlight work presented in OP17 minisymposia on PDE-constrained and risk-averse optimization. Drew Kouri and Thomas Surowiec present their perspective on a growing intersection among optimization, uncertainty quantification, and computational science and engineering.

We welcome Tamás Terlaky's first column as chair of the SIAM Activity Group on Optimization. From Tamás you'll learn the location of OP20. It is also fun to look back and see that OP17 was not the first time that the conference was held in British Columbia. The fifth SIAM Conference on Optimization, OP96, took place in Victoria and was co-chaired by Andy Conn and Margaret Wright.

Many of you have written to opt for an electronic copy of Views and News; for the others among you, please do not hesitate to contact us to opt out of receiving physical copies.

As always, we welcome your feedback, (e-)mailed directly to us or to siagoptnews@lists.mcs.anl.gov. Suggestions for new issues, comments, and papers are always welcome!

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PDE-Constrained Optimization under Uncertainty

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1 Introduction

Uncertainty is pervasive in all science and engineering applications. Incorporating uncertainty in physical models is therefore both natural and vital. In doing so, we often arrive at parametric systems of partial differential equations (PDEs). When passing from simulation to optimization, we obtain (typically nonconvex) infinite-dimensional optimization problems that, upon discretization, result in extremely large-scale nonlinear programs.

For example, consider a linear elliptic PDE on a two-dimensional domain with a single random coefficient. If we sampled the random input with 10,000 realizations of the coefficient, the resulting optimization problem would have 10,000 PDE constraints. Furthermore, discretizing each PDE with piecewise-linear finite elements on a 100×100 uniform quadrilateral mesh results in 100,000,000 degrees of freedom. As a result, the critical components for ensuring mesh-independent performance of numerical optimization methods in the deterministic setting, for example, solution regularity and generalized differentiability, are even more critical in the stochastic setting.

Deterministic PDE-constrained optimization is a well-established discipline within applied mathematics, with a wide array of applications; see, for example, [16, 26, 11]. However, the theory, algorithms, and numerical analysis for optimization problems governed by PDEs with random inputs are a much more recent topic that melds ideas from PDE-constrained optimization with those of stochastic programming and uncertainty quantification, for example, [6, 25, 12, 2, 1, 14, 10].

Parallel to the developments in PDE-constrained optimization, the stochastic programming community has developed theory and algorithms for complex decision-making problems that often exploit specific problem structure, including linearity, convexity, and/or discrete decision variables. Potentially the most important contribution of stochastic programming is the development of mathematical models for uncertainty and risk preference through risk-averse and robust optimization; see, for example, [17, 19, 5, 22]. Nevertheless, little work has been dedicated to the stochastic optimization of distributed parameter systems defined by PDEs with random inputs.

Independent of PDE-constrained optimization and stochastic programming, the field of uncertainty quantification (UQ) has rapidly evolved with the growing need to model variability in complex systems; see [27, 23]. There are, however, shared goals within PDE-constrained optimization and UQ to move beyond forward propagation of uncertainty and solve optimization problems constrained by PDEs with uncertain inputs. In particular, many of the approximation techniques developed in UQ are also relevant discretizations for stochastic PDE-constrained optimization problems.

In this article, we present a perspective on PDE-constrained optimization under uncertainty, including notes on the basic theory, regularization procedures, and several illustrative examples. Our goal is to highlight the differences, special features, and new challenges of these optimization problems when compared with their more familiar deterministic counterparts.

Notation. Throughout this article, $(\Omega, \mathcal{F}, \mathbb{P})$ is a complete probability space; $\mathcal{X} := L^p(\Omega, \mathcal{F}, \mathbb{P})$ for some $p \in [1, \infty]$ is the space of p-integrable random variables on Ω; the physical domain $D \subset \mathbb{R}^n$, $n \in \mathbb{N}$, is an open bounded set with Lipschitz boundary ∂D; the control (decision variable, design, etc.) space Z is a Hilbert space; $Z_{\text{ad}} \subseteq Z$ is a set of feasible controls; and the deterministic state space $U = U(D)$ is a reflexive Banach space.

2 Problem Formulation

Before introducing the general problem formulation, we provide two motivating examples. We start with an example from gas pipeline management.

Example 1 (Optimal Management of a Gas Pipeline [7])**.** The goal of optimal pipeline management is to transmit contracted amounts of gas from supplier to end customer at the lowest possible cost by controlling (i.e., scheduling) compressor activity. Unfortunately, fluctuating pipeline conditions such as future demands (which are affected by weather and market conditions), ambient temperatures at compressor stations leading to variable compression capacity, and equipment availability introduce uncertainty that must be mitigated by any reasonable compressor schedule. For example, a pipeline operator typically does not know far in advance whether a power plant will come online and for how long it will be online.

We can formulate the pipeline management problem as an optimization problem with constraints resulting from, for example, compression capacities and end customer demands. In addition, we can model the flow of gas through the pipeline as a system of nonlinear hyperbolic PDEs. Since the pipeline conditions are uncertain, the solution of this system of PDEs, in other words, the distribution of gas within the pipeline (which is called the line pack), is random. As a result, the objective function, which depends on the line pack, is also random and thus cannot be directly minimized. □

Stochastic PDE-constrained optimization problems also naturally arise in manufacturing applications.

Example 2 (Topological Design with Manufacturing Variability [28])**.** Optical projection lithography is a widely used manufacturing technique in which a design (mask) is transferred onto a substrate via UV light. When building micro- and nano-mechanical devices using optical projection lithography, a number of uncertainties arise due to, for example, the inherent diffraction properties of the lithographic system. As a result, the manufactured device is typically a distorted version of the mask and therefore no longer represents the original design. To account for this, one usually employs a two-step procedure that consists of a blueprint design phase followed by optical proximity correction (OPC). Although OPC accounts for a number of process variations, a residual always remains; and therefore in the worst case the resulting device need not even be functional.

By accounting for the manufacturing variability when formulating the optimal design problem, we can determine a mask that mitigates uncertainty and therefore does not require the two-phase procedure. Aside from constraints accounting for the lithography process and volume restrictions, the mechanical properties of the design are often modeled by using linear elasticity. As in the previous example, the manufacturing variability transfers to the solution of the linear elasticity equations, thus ensuring that the PDE solution is random. Therefore, the objective function to be minimized is a random variable. □

In general, we can represent the governing PDE or system of PDEs by the nonlinear equation

$$e(u, z, \omega) = 0, \tag{1}$$

which we require to hold almost surely (a.s.). That is, a solution u to (1) is required to satisfy (1) only on a subset of Ω with probability equal to one. Here, $z \in Z_{\text{ad}}$ is a control variable and $u(\omega) = [S(z)](\omega)$ is the solution of the equation for fixed z and $\omega \in \Omega$, i.e., u is a random field.

In the deterministic setting (i.e., $S(z)$ is deterministic), our goal is to minimize some objective function J that depends on the PDE solution u, potentially subjected to auxiliary constraints or penalties on z. When the PDE solution is a random field, however, evaluating J at $(S(z), z)$ results in a random variable that cannot be directly minimized. This puts us in the setting of stochastic programming. In particular, any reasonable control should, in some sense, mitigate uncertainty. Since the control z is typically implemented prior to observing the state and its uncertainty, z cannot anticipate this uncertainty; rather, it must be a "good" control for all possible scenarios (i.e., z is deterministic).

Fortunately, decades of research in stochastic programming have produced a multitude of techniques for formulating this problem. For example, we could employ the *robust optimization* approach [4] and solve

$$\min_{z\in Z_{\mathrm{ad}}} \sup_{P\in\mathfrak{A}} \mathbb{E}_P[J(S(z))]$$

where $\mathfrak{A}$ is a subset of probability measures on $(\Omega, \mathcal{F})$. Another possibility would be to use *probabilistic functions* or *stochastic orders* [19, 22]. For example, we might try

$$\min_{z\in Z_{\mathrm{ad}}} \mathbb{P}(\{\omega\in\Omega \,|\, [J(S(z))](\omega)\geq\tau\}),$$

where $\tau\in\mathbb{R}$ is a prescribed threshold. This formulation can be challenging to analyze and solve because of potential nonconvexity and nonsmoothness, despite any nice properties of the solution map $z\mapsto S(z)$ and the objective J. A third option is to incorporate functionals $\mathcal{R}$ known as *risk measures*, which lead to problems with the general structure

$$\min_{z\in Z_{\mathrm{ad}}} \mathcal{R}[\mathcal{J}(S(z))] + \wp(z). \tag{2}$$

Here, $\mathcal{R}:\mathcal{X}\to\overline{\mathbb{R}}:=\mathbb{R}\cup\{+\infty\}$ assigns a numerical value (with the same units) for the random cost $J(S(z))$ and $\wp: Z\to\mathbb{R}$ is the cost of z. Based on duality theory for convex functions, risk-averse optimization is directly related to robust optimization especially in the case when $\mathcal{R}$ is *coherent*.

Many possibilities for $\mathcal{R}$ exist. The most well known are perhaps the expectation (not quite a risk measure per se) and the mean-plus-deviation ($\mathcal{R}=\mathbb{E}+\mathbb{D}$). By axiomatizing risk aversion in the context of mathematical finance, the authors in [3] suggested that a "reasonable" functional $\mathcal{R}$ in (2) should satisfy the following properties.
Let $X, X'\in\mathcal{X}$, $c\in\mathbb{R}$ and $t>0$.

(C1) *Subadditivity*: $\mathcal{R}[X+X']\leq\mathcal{R}[X]+\mathcal{R}[X']$.

(C2) *Monotonicity*: If $X\succeq X'$, then $\mathcal{R}[X]\geq\mathcal{R}[X']$.

(C3) *Translation equivariance*: $\mathcal{R}[X+C]=\mathcal{R}[X]+C$.

(C4) *Positive homogeneity*: $\mathcal{R}[tX]=t\mathcal{R}[X]$.

A functional $\mathcal{R}$ that satisfies these axioms is called a *coherent risk measure*. Although we are not necessarily in the setting of finance or economics, the justifications for using this class are similar for optimal design problems in engineering. For example, (C4) ensures that a change of units of X results in a change of units of $\mathcal{R}[X]$ (i.e., X and $\mathcal{R}[X]$ have the same units). Moreover, (C3) ensures that deterministic quantities, such as the control cost $\wp(z)$, do not affect the risk. In fact, (C3) and (C4) ensure that $\mathcal{R}[c]=c$ for any $c\in\mathbb{R}$.

Perhaps the most popular coherent risk measure is the average or conditional value at risk (CVaR) defined by

$$\mathrm{CVaR}_\beta[X] := \inf_{t\in\mathbb{R}}\left\{t+\frac{1}{1-\beta}\mathbb{E}\left[(X-t)^+\right]\right\},$$

where $\beta\in(0,1)$ and $(\cdot)^+:=\max(0,\cdot)$. Note that whenever the cumulative distribution function (cdf) of X, $F_X(t):=\mathbb{P}(\{\omega\in\Omega\,|\,X\leq t\})$, is continuous, CVaR_β reduces to the upper tail average

$$\mathrm{CVaR}_\beta[X]=\mathbb{E}\left[X|X\geq F_X^{-1}(\beta)\right],$$

where $F_X^{-1}(\beta)$ is the β-quantile of X. Therefore, by using CVaR we are attempting to make decisions z that compensate for large upper tail events.

In any case, the axioms for coherent risk measures (or a subset thereof) enable the development of an existence and optimality theory similar to that of deterministic PDE-constrained optimization. However, difficulties arise due to the potential nonsmoothness of $\mathcal{R}$ and the differentiability properties of $(J\circ S)$. We forego a discussion of sufficient conditions for differentiability and refer the reader to [13]. For now, we set

$$F(z):=\mathcal{J}(S(z))$$

and note that the main difficulty in proving the existence of solutions to (2) lies in demonstrating the weak-lower semicontinuity of the composite objective $(\mathcal{R}\circ F)$. Furthermore, an integral part to deriving optimality conditions for (2) is the subdifferentiability of $\mathcal{R}$ and the Fréchet differentiability of the superposition operator $\mathcal{J}(S(z)):=(J\circ S)(z)$.

With that said, under certain regularity assumptions we can show that (2) indeed has an optimal solution. Furthermore, for any optimal solution $z^\star$ to (2), there exists $\vartheta^\star\in\mathcal{X}^*$ such that

$$\langle\mathbb{E}[\vartheta^\star\nabla F(z^\star)], z-z^\star\rangle+\wp'(z^\star;z-z^\star)\geq 0,\ \forall\, z\in Z_{\mathrm{ad}} \tag{3a}$$
$$\mathcal{R}[X]-\mathcal{R}[F(z^\star)]\geq\mathbb{E}[\vartheta^\star(X-F(z^\star))],\ \forall\, X\in\mathcal{X}. \tag{3b}$$

Using our original notation, we can rewrite (3) in a form that is perhaps more familiar to readers specializing in PDE-constrained optimization. That is, if $z^\star\in Z_{\mathrm{ad}}$ is an optimal solution to (2), then there exists a triple $(u^\star,\lambda^\star,\vartheta^\star)$ such that (3) holds with $\nabla F(z^\star)=e_z(u^\star,z^\star,\cdot)^*\lambda^\star$, where $u^\star$ and $\lambda^\star$ solve

$$e(u^\star,z^\star,\cdot)=0\ \text{ a.s.} \tag{4a}$$
$$e_u(u^\star,z^\star,\cdot)^*\lambda^\star+\mathcal{J}_u(u^\star)=0\ \text{ a.s.} \tag{4b}$$

Here, e_u, e_z, and $\mathcal{J}_u$ denote partial derivatives of e and $\mathcal{J}$. Note that $u^\star$ in (4a) is the solution to the governing PDE whereas the $\lambda^\star$ in (4b) is the solution to the adjoint equation (i.e., the multiplier associated with (4a)) and requires a linearized PDE solve. In addition to the state equation (4a), the adjoint equation (4b), and the usual variational inequality associated with the constraints on z (3a), there is a variational inequality of the second kind (3b) stemming from the definition of the subdifferential of $\mathcal{R}$ at $\mathcal{J}(u^\star)$.

In the "risk-neutral" case (i.e., $\mathcal{R} \equiv \mathbb{E}$), $\vartheta^\star \equiv 1$, and (3b) disappears. In a similar fashion to deterministic PDE-constrained optimization, we could solve the resulting optimality system using a semismooth Newton solver that has been appropriately extended to handle the additional integral over Ω. As mentioned in the introduction, however, the dimension of the discretized problem is significantly larger than that of the deterministic problem. Therefore, special care must be taken when discretizing the random quantities.

An alternative method for solving the risk-neutral problem is the adaptive trust-region algorithm developed in [12]. There, the random PDE is discretized by using adaptive sparse grid collocation. For these optimization problems, we must keep the sparse grid size as small as possible to avoid excess PDE solves. However, we must also ensure that the approximation quality of the sparse grid is sufficient to ensure convergence of the algorithm in the original infinite-dimensional setting. To achieve these goals, the trust-region algorithm employs quadrature error indicators to refine the approximation quality of the objective function value and gradient as the algorithm converges.

Similar to the risk-neutral case, if $\mathcal{R}$ is differentiable at $\mathcal{J}(u^\star)$, then (3b) disappears since $\vartheta^\star = \nabla\mathcal{R}[\mathcal{J}(u^\star)]$. One could then extend the adaptive trust-region approach in [12] to the case of differentiable risk measures by developing the appropriate quadrature error indicators. Nevertheless, coherent risk measures, including CVaR, are in general not differentiable.

3 Regularizing Risk Measures

The discussion in Section 2 suggests that in many cases the nonlinear coupling between (3a) and (3b) may complicate the development of both generalized Newton-type and derivative-based nonlinear programming solvers. One approach to circumventing this difficulty is to regularize or smooth the risk measure and then perform continuation on the associated smoothing parameter. Such techniques are standard in PDE-constrained optimization with constraints on the PDE solution variable.

Ignoring the function space setting and considering a fully discretized problem, we might try to apply an off-the-shelf bundle method for nonconvex, nonsmooth optimization directly. Our next example demonstrates that this approach may not be the best course of action.

Example 3 (Optimal Control of Stationary Viscous Burgers' (see [14])). Given $\alpha = 10^{-3}$ and $D = (0,1)$, consider the stochastic program

$$\min_{z\in L^2(D)} \frac{1}{2}\mathrm{CVaR}_\beta\left[\|S(z)-1\|^2\right] + \frac{\alpha}{2}\|z\|^2, \tag{5}$$

where $\|\cdot\|$ denotes the $L^2(D)$-norm and $u = S(z) : \Omega \to U$ solves (the weak form of) Burger's equation with uncertain inputs:

$$-\nu(\xi)\frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} = f(\xi,\cdot) + z \quad \text{in } D \tag{6a}$$

with boundary conditions

$$u(\cdot,0) = d_0(\xi) \quad\text{and}\quad u(\cdot,1) = d_1(\xi). \tag{6b}$$

Here, $\xi : \Omega \to \Xi = [-1,1]^4$ is a uniformly distributed random vector, and the random coefficients are $\nu(\xi) = 10^{\xi_1-2}$, $f(\xi,x) = \frac{\xi_2}{100}$, $d_0(\xi) = 1 + \frac{\xi_3}{1000}$, and $d_1(\xi) = \frac{\xi_4}{1000}$.

To obtain a finite-dimensional nonlinear program, we discretize in space using continuous piecewise linear finite elements on a mesh of 256 intervals split into three subdomains $\Omega\cup\partial\Omega = [0,0.2]\cup(0.2,0.8)\cup[0.8,1]$. We partition the first subdomain with 80 uniform intervals, the second with 16 uniform intervals, and the third with 160 uniform intervals. We discretize the control analogously. For the random inputs, we use $Q = 10{,}000$ Monte Carlo samples. After discretization, we solve the resulting nonconvex, nonsmooth optimization problem using the trust-bundle method of Schramm and Zowe [21] for three values of the CVaR confidence level $\beta \in \{0.1, 0.5, 0.9\}$. Notice that for any β, we require roughly

Table 1: (Example 3) Number of iterations required by the trust-bundle algorithm to satisfy the prescribed stopping conditions for (5).

β	0.1	0.5	0.9
Iterations	9,740	10,035	10,128

2×10^8 PDE solves to obtain the desired stopping tolerance (see Table 1). This is an unrealistically high number of forward and adjoint solves for a toy example. Moreover, the algorithm terminates at a point that may not satisfy the optimality system (even approximately). The stopping criterion for the trust-bundle algorithm is fulfilled when the norm of the aggregate subgradient, an average of subgradients at previous proximal iterates, is sufficiently small. The relation to the first-order system (3)–(4) is unclear.

In contrast, we can smooth CVaR by using, for example,

$$\mathcal{R}^\beta_\varepsilon[X] := \inf_{t\in\mathbb{R}}\left\{t + \frac{1}{1-\beta}\mathbb{E}\left[(X-t)^+_\varepsilon\right]\right\},$$

where $(\cdot)^+_\varepsilon$ is a C^1-smoothing of the plus function. We can then approximately solve (5) with a sequence of smoothing parameters $\varepsilon_k \downarrow 0$ (assuming small ε is better) using derivative-based optimization methods for each k [14]. Assuming $\varepsilon = 10^{-5}$ results in an acceptable approximation of CVaR, we require roughly 8×10^6 PDE solves to obtain a

Table 2: (Example 3) Behavior of a trust-region algorithm applied in conjunction with smooth CVaR. The numbers in parentheses are the average number of truncated CG iterations per trust-region iteration rounded to the nearest integer.

		$\log(\varepsilon)$					
		0	-1	-2	-3	-4	-5
	0.1	4(13)	4(14)	6(19)	16(6)	26(5)	44(4)
β	0.5	4(17)	4(17)	5(19)	7(22)	7(23)	22(10)
	0.9	5(17)	6(16)	13(13)	16(11)	19(10)	127(3)

control that satisfies a "nearby" first-order system up to the prescribed stopping tolerance of 10^{-8} (see Table 2). That is, smoothing combined with a Newton-type algorithm reduces the computational work by a factor of 25 and produces a solution that is guaranteed to be "nearly" stationary. □

Several possibilities exist for smoothing risk measures. In [14] we use a density-smoothing technique and a dual method that is equivalent to Moreau-Yosida regularization for the case when $\mathcal{R}$ = CVaR. Another possibility is to use infimal convolutions with smooth potential functionals, which we refer to as "epi-regularization" [15]. We briefly describe the latter because it has a number of favorable properties, including the consistency of both minimizers and stationary points (i.e., convergence as $\varepsilon \downarrow 0$) as well as a guaranteed convergence rate. For this approach, we work in a more general class of risk measures than the coherent risk measures. A risk measure $\mathcal{R}$ is said to be *regular* (in the sense of Rockafellar and Uryasev [20]) provided it is proper, closed, and convex and satisfies

(R1) $\mathcal{R}[X] = C$ for all $X \in \mathcal{X}$ such that $X \equiv C \in \mathbb{R}$

(R2) $\mathcal{R}[X] > \mathbb{E}[X]$ for all nondegenerate $X \in \mathcal{X}$.

These are essential minimal requirements that one would expect from any reasonable risk measure. (R1) merely states that something without uncertainty is risk free and (R2) is a means of axiomatizing the concept of risk aversion.

Definition 1 (Epi-Regularized Risk Measures [15]). Let $\mathcal{X} = L^p(\Omega, \mathcal{F}, \mathbb{P})$ with $1 \le p < \infty$, and suppose that $\Phi : \mathcal{X} \to \overline{\mathbb{R}}$ is proper, convex and closed and $\mathcal{R}$ is a regular measure of risk. For $\varepsilon > 0$, we define the epi-regularized risk measure as

$$\mathcal{R}_\varepsilon[X] := \inf_{Y \in \mathcal{X}} \left\{ \mathcal{R}[X - Y] + \varepsilon \Phi\left[\varepsilon^{-1} Y\right] \right\}.$$

Without going into the details, we note that under suitable assumptions one can show that $\mathcal{R}_\varepsilon$ satisfies pointwise error bounds, converges both pointwise and in the sense of Mosco to $\mathcal{R}$, and is consistent; that is, ε-dependent global minimizers converge to global minimizers for the original problem, and ε-dependent stationary points converge to first-order stationary points satisfying a system of the type (3). Furthermore, under local smoothness and convexity properties, one can show that given a solution $z^\star_\varepsilon$ for an epi-regularized problem and $z^\star$ for the original problem, we have

$$\|z^\star_\varepsilon - z^\star\| = O(\sqrt{\varepsilon}). \tag{7}$$

One caveat must be noted, however. To obtain the superior convergence rates (as compared with Monte Carlo) associated with polynomial-based UQ methods (e.g., sparse-grid collocation) for approximating $\mathcal{R}_\varepsilon[F(z)]$, one requires that the random variable to be integrated be sufficiently smooth as a function of ω. As $\varepsilon \downarrow 0$, however, the conditioning of the derivatives of this integrand decreases, leading to performance degradation for polynomial approximations as well as derivative-based optimization algorithms. For example, if $\mathcal{R}_\varepsilon[F(z)]$ is smoothed CVaR, then the integrand is

$$t + \frac{1}{1-\beta}(F(z) - t)^+_\varepsilon,$$

which becomes nonsmooth as $\varepsilon \downarrow 0$.

From our viewpoint, two areas are in need of breakthroughs: (i) the development of discretizations for the underlying random quantities that take into account nonsmooth integrands and (ii) improved algorithms for large-scale nonsmooth, nonconvex optimization (e.g., better continuation strategies for smoothed risk measures).

4 Numerical Experiments

We now solve two examples and discuss what one can achieve by using different risk measures in PDE-constrained optimization under uncertainty. For example, we see how minimizing different risk measures shapes the distributions of $\mathcal{J}(S(z^\star))$. We also compare the cost of a risk-averse solution with that of a risk-neutral one. Our technique for solving these problems is to use epi-regularized risk measures and basic continuation on $\varepsilon \downarrow 0$. Each smooth risk-averse optimization problem is solved by using a trust-region globalization of Newton's method.

Example 4 (Contaminant Mitigation [13]). In this example we consider a simplified model problem representing the risk-averse mitigation of an environmental contaminant. Given nine fixed injection sites, we wish to determine the amount of chemicals to inject at these sites in order to dissolve the contaminant and therefore, minimize the cost of contamination. We model the contaminant transport with a steady advection-diffusion equation.

Let $D := (0,1) \times (0,1)$ and $\partial D := \Gamma_d \cup \Gamma_n$ with $\Gamma_d = \{0\} \times (0,1)$ and $\Gamma_n := \partial D \setminus \Gamma_d$. Let $\kappa_s, \kappa_c > 0$. Then we consider the stochastic program

$$\min_{z \in Z_{\text{ad}}} \mathcal{R}\left[\frac{\kappa_s}{2} \int_D S(z)^2 \,\mathrm{d}x\right] + \kappa_c \|z\|_{\ell^1}, \tag{8}$$

where $S(z) = u : \Omega \to U$ solves the weak form of

$$-\nabla \cdot (\epsilon(\omega) \nabla u) + \mathbb{V}(\omega) \cdot \nabla u = f(\omega) - Bz \quad \text{in } D \tag{9a}$$

with boundary conditions

$$\epsilon(\omega) \frac{\partial u}{\partial n} = 0 \quad \text{on } \Gamma_n \tag{9b}$$

$$u = 0 \quad \text{on } \Gamma_d. \tag{9c}$$

For this problem, the control space is $Z = \mathbb{R}^9$, and we set $Z_{\text{ad}} = \{z \in Z \,:\, 0 \le z \le 1\}$.

The random diffusivity is

$$[\epsilon(\omega)](x) = 0.5 + c^{-1} \exp\left(\delta(x, \omega)\right),$$

where the specific form of δ can be found in [18, Sec. 4, Eqs. 4.2–4] and $c := \max_{(x,\omega)} \exp(\delta(x,\omega)) > 0$. The random advection field $\mathbb{V}$ is

$$[\mathbb{V}(\omega)](x) = \begin{bmatrix} b(\omega) - a(\omega)x_1 \\ a(\omega)x_2 \end{bmatrix},$$

where $b \geq a \geq 0$ $\mathbb{P}$-a.e. and the contaminant source f is the sum of five Gaussian functions whose locations, widths, and magnitudes are random. The deterministic bounded linear operator B is

$$Bz = \sum_{k=1}^{9} z_k \exp\left(-\frac{(x-p_k)^\top (x-p_k)}{2\sigma^2}\right),$$

where $p_k \in (0,1)\times(0,1)$ are the aforementioned control locations and $\sigma = 0.05$.

We discretize the problem using piecewise linear quadrilateral finite elements in space and Monte Carlo with $Q = 1000$ samples for the random inputs. By first replacing each of the random variables with their expected values, we obtain a deterministic problem, which we refer to as the mean-value problem (MVP). The left image in Figure 1 depicts $Bz^\star$, where $z^\star$ solves the MVP. This solution is not risk averse,

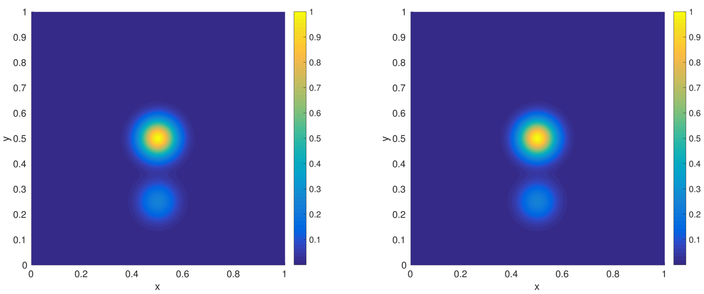

Figure 1: Results for Example 4. $Bz^\star$ for the mean-value problem (left) and the risk-neutral problem (right).

since it ignores the effects of uncertainty. If instead we solve the risk-neutral problem, the resulting optimal control should perform well *on average*. The right image in Figure 1 depicts the risk-neutral optimal control. Note that in this example the solutions to the MVP and the risk-neutral problem are strikingly similar.

On the other hand, if we choose $\mathcal{R}$ to be risk averse, the solution appears to mitigate uncertainty. For example, in the left image in Figure 2 we plot the solution corresponding to the risk measure

$$\mathcal{R}[X] = \inf_{t\in\mathbb{R}} \left\{t + \mathbb{E}[v(X-t)]\right\},$$

where

$$v(x) = (\lambda^{-1}\exp(\lambda x) - 1)_+ - \alpha(-x)_+$$

with $\lambda = 1$ and $\alpha = 0.75$. We introduced and analyzed this risk measure in [13] under the name conditional entropic risk (CER). The behavior of CER can be best understood via the scalar regret function $v(x)$, which ensures that our regret (i.e., displeasure, disutility) grows exponentially for large positive values of x, whereas our regret decreases linearly for negative values of x. We plot the cdfs for five different risk measures (right image in Figure 2). Without specifying the details of each risk measure, we simply note how different risk measures can dramatically influence the shape of the cdf of $\mathcal{J}(S(z^\star))$. For example, while the solutions corresponding to CVaR/CER satisfy

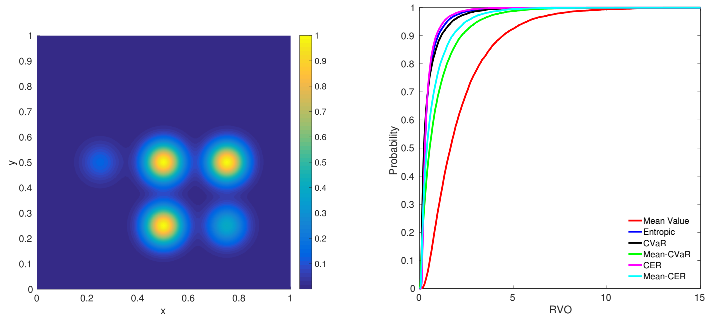

Figure 2: Results for Example 4. $Bz^\star$ obtained by using conditional entropic risk measure (left) and the cdfs of the random variable objectives (RVO) $\mathcal{J}(S(z))$ for various risk measures (right).

$\mathbb{P}(\mathcal{J}(S(z^\star)) \leq 1.5) \approx 0.90$, the solution to the risk-neutral problem satisfies $\mathbb{P}(\mathcal{J}(S(z^\star)) \leq 1.5) \approx 0.35$. $\square$

Example 5 (Semiconductor Doping Optimization)**.** In this example, we investigate the effects of nonlinearity on the ability to shape the distribution of the random objective functional, and we provide experimental evidence for the rate of convergence obtained using epi-regularized risk measures. This problem is related to the robust doping optimization of semiconductor devices, in which one attempts to increase the current flow over the device contacts by introducing impurities (i.e., the dopant) into the wafer. A number of important articles have been written on this topic, we mention here the early work of Fang and Ito [8, 9].

We consider the stochastic program

$$\min_{z\in Z_{\mathrm{ad}}} \mathcal{R}\left[\frac{1}{2}\int_{D_o}(1-S(z))_+^2\,\mathrm{d}x\right] + \frac{\gamma}{2}\int_D z^2\,\mathrm{d}x, \qquad (10)$$

where $D = (0, 0.6) \times (0, 0.2)$, $D_o = (0.5, 0.6) \times (0.167, 0.2)$, $\gamma = 10^{-2}$, and $u = S(z) : \Omega \to U$ solves the weak form of the semilinear elliptic PDE

$$-\kappa(\xi)\Delta u + c(\xi)\sinh(u) = B(\xi)z + b \quad \text{in } D$$

with boundary conditions

$$\kappa(\xi)\frac{\partial u}{\partial n}(\cdot, x) = 0 \quad \text{on } \partial D.$$

The operator B satisfies $d = B(\xi)z : \Omega \to U$ and is the weak solution to the linear elliptic PDE

$$-r(\xi)\Delta d + d = z \quad \text{in } D$$

with boundary conditions

$$r(\xi)\frac{\partial d}{\partial n} = 0 \quad \text{in } \partial D.$$

Here, we set

$$\kappa(\xi) = 2.5\times 10^{\xi_1},\ c(\xi) = 1.45\times 10^{\xi_2},\ r(\xi) = 10^{\xi_3},$$

where ξ_1 is uniformly distributed on $[-2,-1]$, ξ_2 is uniformly distributed on $[-1,0]$, and ξ_3 is uniformly distributed on $U[-4,-1]$, and

$$b(x) = 12 \cdot \mathbb{1}_{D_b}(x)$$

where $D_b = (0, 0.1)\times(0.167, 0.2)$ and $\mathbb{1}_{D_b}$ denotes the characteristic function of the set D_b. We discretize both PDEs using piecewise linear quadrilateral finite elements, and we approximate the random inputs $\xi = (\xi_1, \xi_2, \xi_3)$ using $Q = 1000$ Monte Carlo samples. Furthermore, we set $\mathcal{R} \equiv \mathrm{CVaR}_\beta$ and use $\Phi[X] = \frac{1}{2}\mathbb{E}[X^2] + \mathbb{E}[X]$ for the epi-regularization.

In Figures 3 and 4, we see that with increasing CVaR confidence level β, the variability in the random objective decreases as expected. Furthermore, the random variables $\mathcal{J}(S(z^\star))$ for larger β stochastically dominate those for smaller β. Despite a lack of provable convexity for the optimization problem, we still observe the theoretical convergence rates for the computed optimal controls versus the optimal solution in Figure 4. □

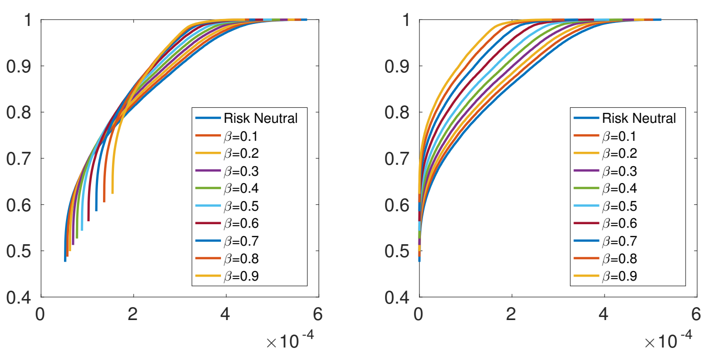

Figure 3: Results for Example 5. Left: Cumulative distribution function for the random objective plus control penalty evaluated at the optimal controls. Right: Cumulative distribution function for the random objective evaluated at the optimal controls.

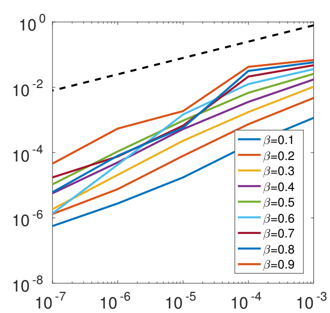

Figure 4: Results for Example 5. Epi-regularization error in the optimal controls for $\beta \in \{0.1, 0.2, \ldots, 0.9\}$ and $\varepsilon \in \{10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}\}$. The dashed line depicts the theoretical convergence rate of $\frac{1}{2}$.

5 Outlook

A vast array of challenges and open problems exists within the field of PDE-constrained optimization under uncertainty, including the development of computationally efficient methods for solving risk-averse PDE-constrained optimization problems that are sample independent or weakly sample dependent and the development of adaptive methods for cases other than risk neutral. Other open areas include extending modern UQ methods for approximating PDEs with uncertain inputs, such as stochastic collocation, polynomial chaos and other spectral projection methods, active subspaces, and low-rank tensor approximation, to solve the corresponding optimization problems constrained by such PDEs.

Aside from numerical issues, open problems exist in formulating stochastic optimization problems constrained by transient PDEs. For example, how are time-dependent uncertainties modeled and how are time-dependent controls/decisions implemented? In this case, the issue of time-consistent risk measures for continuous-time systems looms large. Recently, some researchers (e.g., [24]) have formulated such dynamic optimal control problems using the Fokker-Planck equation, which transforms the governing stochastic differential equation into a deterministic PDE whose solution is the probability density of the stochastic state variable. This approach is attractive; however, it requires the discretization of PDEs with potentially enormous spatial dimension.

It is unclear how to formulate state constraints (e.g., constraints on displacement or strain in elasticity) when the PDE solution is uncertain. One promising, albeit challenging, approach is to enforce state constraints through stochastic dominance, which has clear ramifications in terms of both theory and numerics.

Acknowledgments. DPK's research was sponsored by DARPA EQUiPS grant SNL 014150709. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

TMS's research was sponsored by the DFG grant no. SU 963/1-1 "Generalized Nash Equilibrium Problems with Partial Differential Operators: Theory, Algorithms, and Risk Aversion."

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Bulletin

1 Event Announcements

1.1 2018 Mixed Integer Programming Workshop (MIP 2018)

The 2018 workshop in Mixed Integer Programming (MIP 2018) will be held June 18–21 at Clemson University (Greenville, South Carolina). The 2018 Mixed Integer Programming workshop will be the fifteenth in a series of annual workshops held in North America designed to bring the integer programming community together to discuss very recent developments in the field. The workshop consists of a single track of invited talks and features a poster session that provides an additional opportunity to share and discuss recent research in MIP. Registration details, a call for participation in the poster session, and information about student travel awards will be made in a subsequent announcement.

Confirmed Speakers: Bob Bixby, Gurobi; Chen Chen, Columbia University & Ohio State University; Grard Cornujols, Carnegie Mellon University; Yuri Faenza, Columbia University; Ricardo Fukasawa, University of Waterloo; Matthew Galati, SAS; Andres Gomez, University of Pittsburgh; Aida Khajavirad, Carnegie Mellon University; Pierre Le Bodic, Monash University; Quentin Louveaux, Universit de Lige; Marco Lbbecke, RWTH Aachen; Miles Lubin, Google; Stephen Maher, Lancaster University; Enrico Malaguti, Universit di Bologna; Jim Ostrowski, University of Tennessee; Joe Paat, ETH Zurich; Annie Raymond, University of Washington; Suvrajeet Sen, University of Southern California; David Shmoys, Cornell University; Cole Smith, Clemson University; Wolfram Wiesemann, Imperial College.

More details are available on the workshop website: `https://or.clemson.edu/mip-2018/`.

1.2 23rd International Symposium on Mathematical Programming (ISMP 2018)

The 23rd International Symposium on Mathematical Programming (ISMP 2018) will take place in Bordeaux, France, July 1-6, 2018. The organizers have the great pleasure of inviting you to prepare contributions to this world congress of mathematical optimization gathering scientists as well as industrial researchers and users of mathematical optimization. Plenary, semi-plenary, and keynote speakers for ISMP2018 have been announced on the conference website along with their provisional talk titles: `https://ismp2018.sciencesconf.org/resource/page/id/2`.

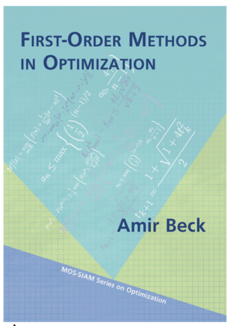
The richness of the conference scientific program shall be further enhanced by the attendees' contributions that are organized in parallel sessions by the scientific committee. These contributions can take the forms of mini-symposium, invited sessions and contributed presentations. Each attendee can submit at most one talk. Candidate session organizers are invited to contact the scientific committee for their particular stream of interest.

Important Dates
February 28, 2018: Abstract submission deadline
April 30, 2018: Early bird registration deadline and registration deadline for presenting authors

More details are available on the conference website: https://ismp2018.sciencesconf.org/

2 Book Announcements

2.1 First-Order Methods in Optimization



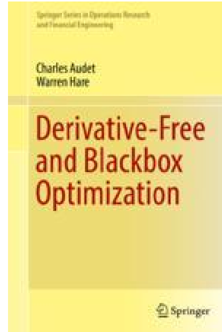
By Amir Beck
Publisher: SIAM
Series: MOS-SIAM Series on Optimization, Vol. 25
ISBN: 978-1-611974-98-0, x + 484 pages
Published: 2017
http://bookstore.siam.org/mo25/

About the book: The primary goal of this book is to provide a self-contained, comprehensive study of the main first-order methods that are frequently used in solving large-scale problems. First-order methods exploit information on values and gradients/subgradients (but not Hessians) of the functions composing the model under consideration. With the increase in the number of applications that can be modeled as large or even huge-scale optimization problems, there has been a revived interest in using simple methods that require low iteration cost as well as low memory storage. The author has gathered, reorganized, and synthesized (in a unified manner) many results that are currently scattered throughout the literature, many of which cannot be typically found in optimization books.

First-Order Methods in Optimization offers comprehensive study of first-order methods with the theoretical foundations; provides plentiful examples and illustrations; emphasizes rates of convergence and complexity analysis of the main first-order methods used to solve large-scale problems; and covers both variables and functional decomposition methods.

Audience: This book is intended primarily for researchers and graduate students in mathematics, computer sciences, and electrical and other engineering departments. Readers with a background in advanced calculus and linear algebra, as well as prior knowledge in the fundamentals of optimization (some convex analysis, optimality conditions, and duality), will be best prepared for the material.

2.2 Derivative-Free and Blackbox Optimization



By Charles Audet and Warren Hare
Publisher: Springer
Series: Operations Research and Financial Engineering
ISBN: 978-3-319-68913-5, xviii + 302 pages
Published: 2017
http://www.springer.com/us/book/9783319689128

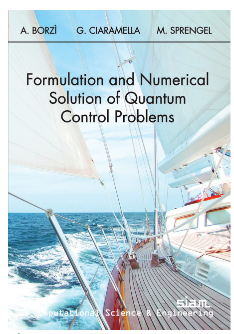
About the book: This book is designed as a textbook, suitable for self-learning or for teaching an upper-year university course on derivative-free and blackbox optimization.

The book is split into 5 parts and is designed to be modular; any individual part depends only on the material in Part I. Part I of the book discusses what is meant by Derivative-Free and Blackbox Optimization, provides background material, and early basics while Part II focuses on heuristic methods (Genetic Algorithms and Nelder-Mead). Part III presents direct search methods (Generalized Pattern Search and Mesh Adaptive Direct Search) and Part IV focuses on model-based methods (Simplex Gradient and Trust Region). Part V discusses dealing with constraints, using surrogates, and bi-objective optimization.

End of chapter exercises are included throughout as well as 15 end of chapter projects and over 40 figures. Benchmarking techniques are also presented in the appendix.

Audience: Flexible usage suitable for undergraduate, graduate, mathematics, computer science, engineering, or mixed classes.

2.3 Formulation and Numerical Solution of Quantum Control Problems



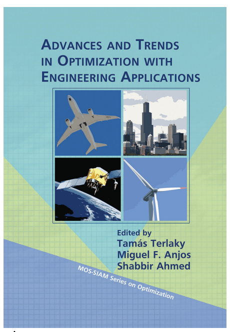
By Alfio Borzì, Gabriele Ciaramella, and Martin Sprengel
Publisher: SIAM
Series: Computational Science and Engineering, Vol. 16
ISBN: 978-1-611974-83-6, x + 390 pages
Published: 2017
http://bookstore.siam.org/cs16/

About the book: This book provides an introduction to representative nonrelativistic quantum control problems and their theoretical analysis and solution via modern computational techniques. The quantum theory framework is based on the Schrdinger picture, and the optimization theory, which focuses on functional spaces, is based on the Lagrange formalism. The computational techniques represent recent developments that have resulted from combining modern numerical techniques for quantum evolutionary equations with sophisticated optimization schemes. Both finite and infinite-dimensional models are discussed, including the three-level Lambda system arising in quantum optics, multispin systems in NMR, a charged particle in a well potential, BoseEinstein condensates, multiparticle spin systems, and multiparticle models in the time-dependent density functional framework.

This self-contained book covers the formulation, analysis, and numerical solution of quantum control problems and bridges scientific computing, optimal control and exact controllability, optimization with differential models, and the sciences and engineering that require quantum control methods.

Audience: This book is intended for mathematicians working on ODE/PDE control and optimization problems and the numerical analysis of differential equations; physicists; chemists; and engineers who focus on quantum control problems. It is suitable for advanced courses on ODE/PDE quantum control problems and provides extensively elaborated problems that help the reader develop insight into the main ideas and techniques of quantum control problems. It is also suitable for advanced graduate students and scientists of mathematics, the natural sciences, and engineering.

2.4 Advances and Trends in Optimization with Engineering Applications



Editors: Tamás Terlaky, Miguel F. Anjos, and Shabbir Ahmed
Publisher: SIAM
Series: MOS-SIAM Series on Optimization, Vol. 24
ISBN: 978-1-611974-67-6, xxxiv + 696 pages
Published: 2017
http://bookstore.siam.org/mo24/

About the book: Optimization is of critical importance in engineering. Engineers constantly strive for the best possible solutions, the most economical use of limited resources, and the greatest efficiency. As system complexity increases, these goals mandate the use of state-of-the-art optimization techniques.

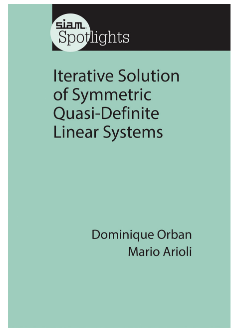
In recent years, the theory and methodology of optimization have seen revolutionary improvements. Moreover, the exponential growth in computational power, along with the availability of multicore computing with virtually unlimited memory and storage capacity, has fundamentally changed what engineers can do to optimize their designs. This is a two-way process: engineers benefit from developments in optimization methodology, and challenging new classes of optimization problems arise from novel engineering applications.

Advances and Trends in Optimization with Engineering Applications reviews 10 major areas of optimization and related engineering applications, providing a broad summary of state-of-the-art optimization techniques most important to engineering practice. Each part provides a clear overview of a specific area and discusses a range of real-world problems.

The book provides a solid foundation for engineers and mathematical optimizers alike who want to understand the importance of optimization methods to engineering and the capabilities of these methods.

Audience: This book will be of interest to doctoral students, recent graduates, experienced researchers, and practitioners in engineering, optimization, and operations research.

2.5 Iterative Solution of Symmetric Quasi-Definite Linear Systems



By Dominique Orban and Mario Arioli
Publisher: SIAM
Series: SIAM Spotlights, Vol. 3
ISBN: 978-1-611974-72-0, xiv + 93 pages
Published: 2017
http://bookstore.siam.org/sl03/

About the book: Numerous applications, including computational optimization and fluid dynamics, give rise to block linear systems of equations said to have the quasi-definite structure. In practical situations, the size or density of those systems can preclude a factorization approach, leaving only iterative methods as the solution technique. Known iterative methods, however, are not specifically designed to take advantage of the quasi-definite structure.

This book discusses the connection between quasi-definite systems and linear least-squares problems, the most common and best understood problems in applied mathematics, and explains how quasi-definite systems can be solved using tailored iterative methods for linear least squares (with half as much work!). To encourage researchers and students to use the software, it is provided in MATLAB, Python, and Julia.

The authors provide a concise account of the most well-known methods for symmetric systems and least-squares problems, research-level advances in the solution of problems with specific illustrations in optimization and fluid dynamics, and a website that hosts software in three languages.

Audience: This book is intended for researchers and advanced graduate students in computational optimization, computational fluid dynamics, computational linear algebra, data assimilation, and virtually any computational field in which saddle-point systems occur. The software should appeal to all practitioners, even those not technically inclined.

3 Other Announcements

3.1 2017 SIAM Fellows Announced

Each year, SIAM designates as Fellows of the society those who have made outstanding contributions to the fields of applied mathematics and computational science. This year, 28 members of the community were selected for this distinction.

These new Fellows include eight members of the SIAG, whose citations are included below. Full details on the SIAM Fellow program can be found at `http://www.siam.org/prizes/fellows/index.php`. Congratulations to all the new Fellows!

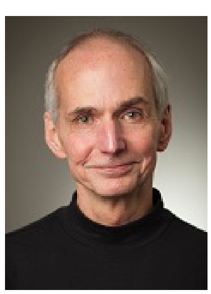
Rama Cont
Imperial College London
For contributions to financial mathematics and stochastic analysis.

Bart De Moor
KU Leuven
For contributions to concepts and algorithms in numerical multilinear algebra and applications in engineering.

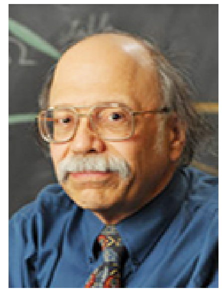
Andreas Griewank
Yachay Tech University, School of Mathematical Sciences and Information Technology
For fundamental contributions to algorithmic differentiation and to iterative methods for nonlinear optimization.

Monique Laurent
Centrum Wiskunde & Informatica (CWI), Amsterdam, and Tilburg University
For contributions to discrete and polynomial optimization and revealing interactions between them.

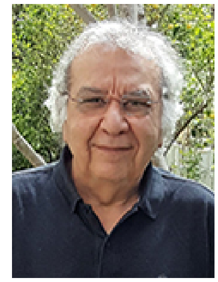
Lois Curfman McInnes
Argonne National Laboratory
For contributions to scalable numerical algorithms and software libraries for solving large-scale scientific and engineering problems.



James Renegar
Cornell University
For fundamental results on continuous optimization and the interface between algorithms, numerical analysis, and algebra.



Andrew Sommese
University of Notre Dame
For foundational contributions to the numerical solution of polynomial systems and applications of algebraic geometry.



Marc Teboulle
Tel Aviv University
For fundamental contributions to continuous optimization theory, analysis, development of algorithms, and scientific applications.

3.2 Beale-Orchard-Hays Prize

Nominations are invited for the 2018 Beale-Orchard-Hays Prize for Excellence in Computational Mathematical Programming. The Prize is sponsored by the Mathematical Optimization Society, in memory of Martin Beale and William Orchard-Hays, pioneers in computational mathematical programming. Nominated works must have been published between Jan 1, 2012 and Dec 31, 2017, and demonstrate excellence in any aspect of computational mathematical programming. "Computational mathematical programming" includes the development of high-quality mathematical programming algorithms and software, the experimental evaluation of mathematical programming algorithms, and the development of new methods for the empirical testing of mathematical programming techniques. Full details of prize rules and eligibility requirements can be found at http://www.mathopt.org/?nav=boh.

Nominations can be submitted electronically or in writing, and should include detailed publication details of the nominated work. Electronic submissions should include an attachment with the final published version of the nominated work. If done in writing, submissions should include five copies of the nominated work. Supporting justification and any supplementary material are strongly encouraged but not mandatory. The prize committee reserves the right to request further supporting material and justification from the nominees. The deadline for nominations is January 15, 2018. Nominations should be submitted to Dr. Michael Grant (mcg@cvxr.com). If you wish to submit a nomination in writing, please contact Dr. Grant for a mailing address.

Chair's Column

This is the first issue of SIAG/OPT Views and News in which I have the good fortune to welcome you. I am deeply honored to have been elected as chair of the SIAM Activity Group of Optimization. I am looking forward to exciting and productive years for SIAG/OPT. The other new officers are Andreas Waechter (vice chair), Michael Friedlander (program director), and Jim Luedtke (secretary/treasurer); and I am pleased to inform you that Jennifer Erway and Stefan Wild generously agreed to continue as newsletter editors.

Before turning to other news, I offer my thanks to the previous leadership team for their great work. The team of Juan Meza, Martine Labbe, Michael Friedlander, and Kim-Chuan Toh further strengthened SIAG/OPT, evident by the monotonically growing membership and the outstanding OP'17 conference. As you see, this edition is devoted to a highlight of the highly successful and record-breaking OP'17 in Vancouver, May 2017. OP'17 featured seven plenary presentations by Eva Lee, Jeffrey Linderoth, Zhi-Quam Luo, Ali Pinar, James Renegar, Katya Scheinberg, and Martin Wainwright and covered a range of topics, included deep algorithmic and complexity issues, applications, and emerging areas such as optimization in machine learning. The two minitutorials by Pascal Van Hentenryck and Daniel Bienstock (Optimal Power Flow) and by Francis Bach and Mark Schmidt (Stochastic Optimization for Machine Learning) demonstrated both the power of optimization to solve relevant problems, and the community's interest in emerging areas of research. This landmark conference attracted an unprecedented 709 participants. The continued growth of our membership and size of our conferences proves the growing popularity and relevance of optimization and the strength of our community. The conference offered ample opportunities to meet old and new friends, to discuss research, and to further strengthen the fabric of our great community.

Talking about successes and notable achievements, indeed, we had a lot to celebrate. Several SIAG/OPT colleagues were selected as SIAM Fellows: Gang Bao, Thomas Coleman, Michael Hintermüller, Andrew Knyazev, James Nagy, Cynthia Phillips, and David Williamson in 2016; Rama Cont, Bart De Moor, Andreas Griewank, Monique Laurent, Lois McInnes, James Renegar, Andrew Sommese, and Marc Teboulle in 2017. I encourage all of you to take the time to nominate our exceptional colleagues for SIAM's Distinguished Fellows classes. I also congratulate the 2017 SIAG/Optimization prize winners Jérôme Bolte, Shoham Sabach, and Marc Teboulle. On behalf of the winning team, Shoham Sabach from Technion Israel Institute of Technology gave an excellent lecture on their paper "Proximal Alternating Linearized Minimization for Nonconvex and Nonsmooth Problems." Congratulations to all Fellows and winners!

Our membership in numbers is healthy, well over 1,150 total members. With small variation, the number of nonstudent members holds steady around 570, while the number of student members increased significantly from 386 in 2014 to 613 in 2017. I hope that all our members remain dedicated members of our SIAG/OPT for many years to come, and you that share our enthusiasm and encourage students, friends, and colleagues to sign up for the Optimization Activity Group of SIAM. Please also remember that students at many of our universities receive free membership, and it's a great way for them to get to know SIAM and to boost their professional network through SIAG/OPT.

You may already be thinking of attending OP'20, our next optimization conference. In Vancouver, during the business meeting, we had a presentation by colleagues from Hong Kong about the opportunity to have OP'20 at Hong Kong Polytechnic University. The proposal was well prepared and well received. After OP'17 we and the SIAM office had further details of the proposal clarified and decided that the 2020 SIAM Conference on Optimization will be held May 26–29, 2020 at Hong Kong Polytechnic University. The local organizing committee co-chairs are Xiaojun Chen and Defeng Sun, and I will have the honor of serving as conference co-chair with Defeng Sun. This structural overlap will ensure seamless collaboration and information flow between the two committees. All this has been approved by SIAM, so we are looking forward to another celebration of the advances of optimization research at OP'20 in Hong Kong!

Another important action item from the Vancouver business meeting was to explore the possibility of establishing another SIAM Optimization Prize. The attendees supported the initiative, noting that other SIAGs already have student or early-career prizes and that having another prize would be good PR for SIAG/OPT and would enable us to recognize emerging stars in optimization. During the summer the leadership team regularly met (by Skype, phone, and e-mails) to discuss what the second SIAM Optimization Prize should be. We developed a proposal to establish an early-career prize, and the proposal is in SIAM's hands to verify for consistency with other SIAM prizes and to approve or request modification.

I have another request. Recently, our program director, Michael Friedlander, announced that SIAG/OPT has been asked to sponsor a set of minisymposia for the SIAM Annual Meeting (AN18), to be held in Portland, OR, July 9–13, 2018. Minisymposium proposals should be sent to Michael Friedlander (mpf@cs.ubc.ca) by January 3, 2018. Please contribute to a strong optimization presence at AN18!

I wish all of you a wonderful holiday season, and a very happy and productive 2018!

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