Pathfinding Thermodynamically Reversible Quantum Computation

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Outline

- Landauer's limit and the quest for reversible computing.
- Resource theory of quantum thermodynamics.
- Generalized reversible computing and dissipation bound on quantum computing.
- Modelling in quantum open systems.
- State space geometric properties of open systems and impact on generalized reversible computing.
- Application to the dissipation delay product.
- Asynchronous ballistic quantum computing.

Landauer's Limit

- *Landauer's limit*[1]: one bit of information lost in computational process dissipates $\Delta E \geq k_B T \ln 2$ of energy as heat.
  - Dissipation due to increase in entropy: $\Delta S \geq k_B \ln 2$. Links information and physics!
  - Ejection of information in correlated bits[2]: loss of prior correlations to environment.
    - Ejection of *uncorrelated* bits to the environment does *not* contribute to change in entropy.
- No-hiding theorem[3]: information can't be destroyed.
  - Moves from system S to environment E. Global unitary evolution over $\mathcal{H}_S \otimes \mathcal{H}_E$.
  - Information lost from original system can't remain in SE correlations.
- Conventional computing: *entirely* irreversible. (Ex.: clearing memory.)

[1] – R. Landauer, IBM J. Res. Dev. **5**, 163 (1961).
[2] – M. Frank, arXiv:1806.10183.
[3] – S. Braunstein and A. K. Pati, Phys. Rev. Lett. **98**, 080502 (2007).

Resource Theory

- *Resource theory*[4]: all *free states* $\hat{\rho} \in \mathcal{D}(\mathcal{H})$ and *free operations* $\Phi \in \mathrm{Aut}\big(\mathcal{D}(\mathcal{H})\big)$ which can be implemented with no energetic cost or dissipation, with conditions.
  - Ex.: Resource theory of bipartite entanglement. Free states: separable states. Free operations: local unitaries & classical communication (LOCC). Necessary & sufficient condition: $|\psi\rangle\langle\psi|_{AB} \mapsto |\phi\rangle\langle\phi|_{AB}$ iff $S(\psi_A) \geq S(\phi_A)$.
- Resource theory of quantum thermodynamics (RTQT):
  - Free states for nontrivial $\beta\hat{H}$: equilibrium thermal states $\{\hat{\rho}_G\}$.
  - Free operations for nontrivial $\beta\hat{H}$: all operations that preserve $\{\hat{\rho}_G\}$. *Thermal operations*.
  - Condition for $\hat{\rho} \mapsto \hat{\sigma}$ under thermal operations: *thermomajorization.*

[4] – N. H. Y. Ng and M. P. Woods, in *Thermodynamics in the Quantum Regime*, edited by F. Binder *et al.* (Springer Nature, Cham, 2018); see also arXiv:1805.09564.

Thermal Operations and Thermomajorization

- Start: system S coupled to environment E. E starts in thermal state: $\hat{\rho}_{E,G}$.
- <u>Thermal operations</u>: all quantum channels $\mathcal{E}(\hat{\rho}_S)$: $\hat{\rho}_S \mapsto \mathrm{Tr}_E \left\{ \hat{U}_{SE} \left(\hat{\rho}_S \otimes \hat{\rho}_{E,G} \right) \hat{U}_{SE}{}^{\dagger} \right\}$.
  - Energy-conserving unitary dynamics: global unitary evolution $\hat{U}_{SE}$ across SE.
  - Require $\left[\hat{U}_{SE}, \hat{H}_S + \hat{H}_E \right] = 0$ at all times.
  - Maps thermal states to thermal states (but not necessarily the same one).
- <u>Thermomajorization</u>: necessary & sufficient for $\mathcal{E}(\hat{\rho})$ to map $\hat{\rho}_S \mapsto \hat{\sigma}_S$. For eigenvalues $\lambda_i(\hat{\rho})$ of $\hat{\rho}$ with corresponding energies $E_{i,\hat{\rho}}$, require:

$$\left(\sum_i e^{-\beta E_{i,\hat{\rho}}}, \sum_i \lambda_i(\hat{\rho})^{\downarrow} \right) \geq \left(\sum_i e^{-\beta E_{i,\hat{\sigma}}}, \sum_i \lambda_i(\hat{\sigma})^{\downarrow} \right)$$

  - $\lambda_i(\hat{\rho})^{\downarrow}$: order eigenvalues of $\hat{\rho}$ by decreasing value.

Classical Computing as a Lower Dissipative Bound

- Information processing expressed as a thermal operation[5]. Dissipation:

$$\Delta E_Q \geq k_B T\left(S(\hat{\rho}_S) - S(\hat{\varrho}_S)\right) + S\left(\hat{U}_{SME}\left(\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)\hat{U}_{SME}{}^{\dagger} \middle\| \hat{\varrho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)$$

  - System S coupled to environment E and catalyst M; same as splitting E into M and E.
  - Channel: $\mathcal{E}(\hat{\rho}_S): \hat{\rho}_S \mapsto \hat{\varrho}_S \coloneqq \mathrm{Tr}_M\,\mathrm{Tr}_E\left\{\hat{U}_{SME}\left(\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)\hat{U}_{SME}{}^{\dagger}\right\}$.
  - First term: information cost of classical IP. Second term: quantum IP.

- Classical IP is a *lower* dissipative bound! Quantum IP can be equal at best.
  - Classical IP: signal states correspond to orthogonal quantum states.
  - Pure unitaries and single input & output operations match classical IP dissipation bound.

[5] – D. Bedingham and O. Maroney, New J. Phys. **18**, 113050 (2016)

Generalized Reversible Computing

- *Reversible computing*: reversibly transform computational states, instead of destructively overwriting them.
  - Computational states c: states representing computational information.
    - $\{c\}$ partitions $\{\hat{\rho}\}$ in equiv. classes. All $\{\hat{\rho}\}$ in same class are linked by unitary transform: have same entropy.
  - GRC: Bijections on the probability-1 subset of $\{c\}$.
  - Lower dissipative bound on classical RC can be zero!
    - Quantum RC: bounded only by mutual entropy term.

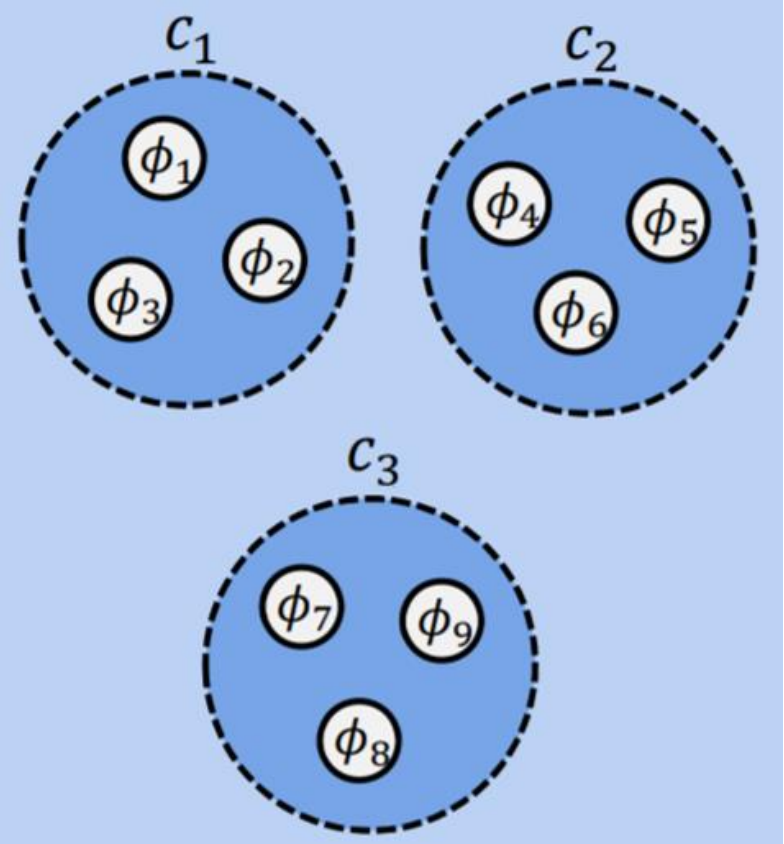


Image modified from M. Frank, arXiv:1806.10183.

Resource Theory & Quantum Reversible Computing

- Key issue: IP operations usually involve a catalyst machine, which sometimes needs to be reset. Reset destroys correlations (mutual info.): dissipation!
  - This is also the source of dissipation in irreversible (classical) computing.
- Thermal operations *can* provide a framework for quantum RC with dissipation arbitrarily close to classical RC.
  - Same idea as classical RC: preserve correlations. Rigorously proved[6]: examine the transition $\hat{\rho}_S \otimes \hat{\rho}_M \otimes |w\rangle\langle w| \mapsto \hat{\varrho}_{SM} \otimes |w-\Delta\rangle\langle w-\Delta|$ for SME coupled to work bit $|w\rangle$. Resetting condition: $\mathrm{Tr}_S\, \hat{\varrho}_{SM} = \hat{\rho}_M$.
  - $S(\hat{\rho}_S \| \mathrm{Tr}_S\, \hat{\varrho}_{SM})$ can be arbitrarily close to zero: arbitrarily close to classical IP bound.

[6] – M. Müller, Phys. Rev. X **8**, 041051 (2018).

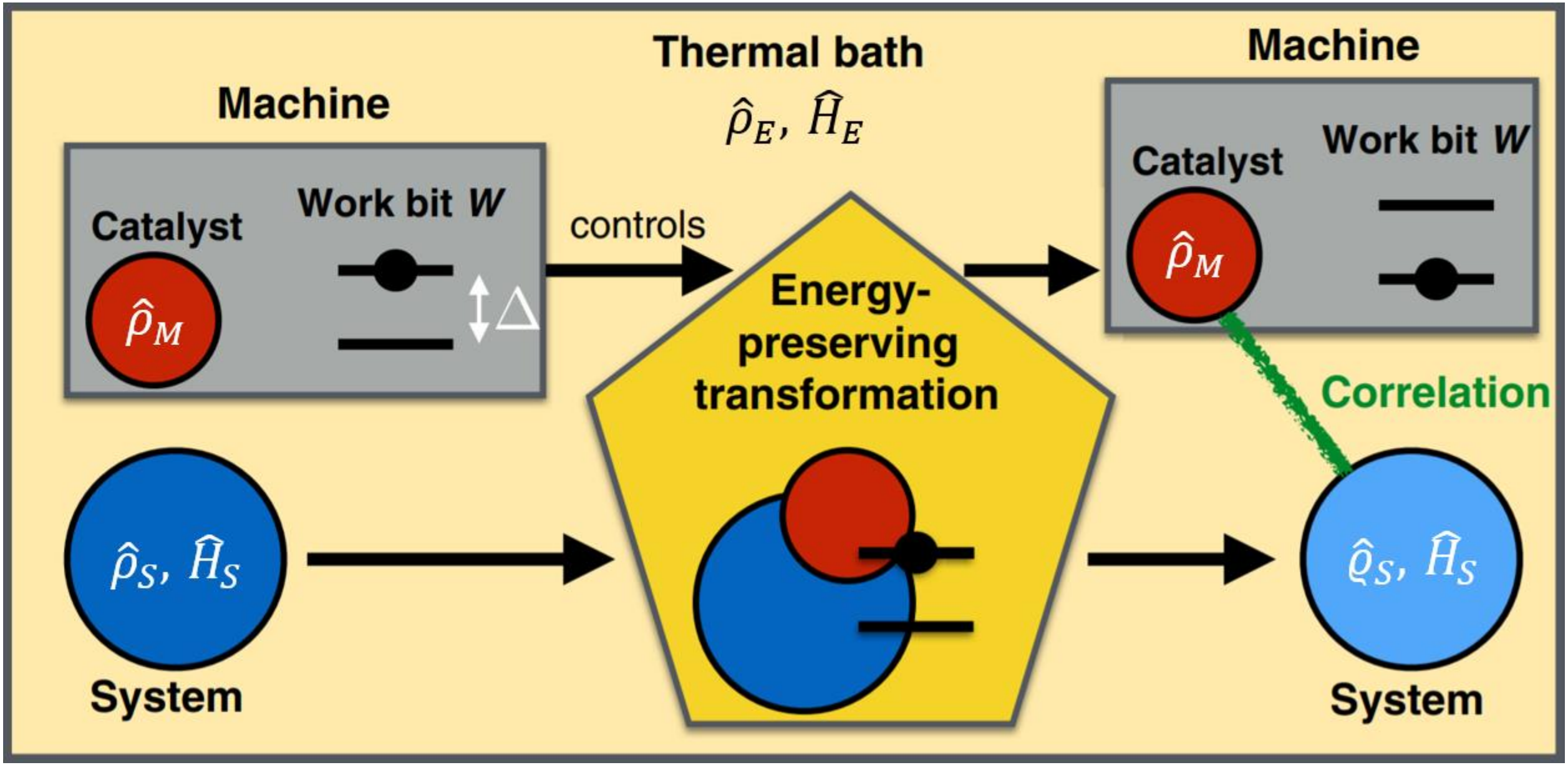


Image taken from M. Müller, Phys. Rev. X **8**, 041051 (2018).

Modelling Reversible Computational Processes

- Standard[7] method: N density matrices encoding N-ary alphabet.
  - Bijection between physical and computational states.
  - Generalized classical reversible computing: more physical states than computational states.
- Need to generalize to GRC, thermal operations, and nonequil. dynamics.
  - Want physical state $\rightarrow$ computational state mapping to always commute with $\hat{U}_{SE(MW)}$. The same physical state always maps to the same computational state.
  - Can permit long-lived quantum coherences between physical states within an equiv. class.
  - Want to represent these as thermal operations, possibly within a larger open system.

[7] – N. Anderson, Eur. Phys. J. B **91**, 156 (2018).

(System) Kraus Operators

- ∞-simal evolution for density matrices in open (noneq.) quantum systems:

$$\mathcal{E}_{\mathrm{d}t}[\hat{\rho}_S(t)] = \hat{\rho}_S(t+\mathrm{d}t) = \sum_{jk} \langle e_k | \hat{U}_{SE}(\mathrm{d}t) | e_j \rangle \, \hat{\rho}_S(t) \, \langle e_j | \hat{U}_{SE}(\mathrm{d}t) | e_k \rangle$$

  - $\hat{E}_{jk} \coloneqq \sqrt{\lambda_j} \langle e_k | \hat{U}_{SE} | e_j \rangle$: (system) Kraus operators over environment eigenstates $\{|e_i\rangle\}$.
  - Any map of the form $\hat{\rho}_S(t) \mapsto \sum_{jk} \hat{E}_{jk}(t) \, \hat{\rho}_S(0) \, \hat{E}_{jk}{}^{\dagger}(t)$ is a *CPTP map.*
  - $\hat{E}_{jk} \in \mathsf{Op}(\mathcal{H}_S)$: takes $\mathcal{D}(\mathcal{H}_S)$ to itself, but expression depends on $\{|e_i\rangle\}$.
    - Done via algebra operation $\mathsf{Op}(\mathcal{H}_S) \times \mathsf{Op}(\mathcal{H}_S) \to \mathsf{Op}(\mathcal{H}_S)$. $\hat{E}_{jk} \notin \mathsf{L}_1\big(\mathsf{Op}(\mathcal{H}_S)\big)$.

The GKSL Superoperator

- GKSL superop.: time evolution generator at lowest order. $\mathcal{E}_{\mathrm{d}t} = \mathcal{I} + \mathrm{d}t\,\mathcal{L} + \cdots$, and $\mathcal{L} \coloneqq \lim_{\mathrm{d}t \to 0} (\mathcal{E}_{\mathrm{d}t} - \mathcal{I})/\mathrm{d}t$.
- Kraus operators give evolution of $\hat{\rho}$ under quantum jump:

$$\hat{E}_0 \coloneqq \mathbb{1}_S - i\hat{H}_S\,\mathrm{d}t - \frac{1}{2}\sum_{j,k>0} \hat{F}_{jk}{}^{\dagger}\hat{F}_{jk}\,; \quad \hat{E}_{j,k>0} = \sqrt{\kappa_{jk}\,\mathrm{d}t}\,\hat{F}_{jk}$$

- GKSL superoperator in terms of jump operators $\{\hat{F}_{jk}\}$:

$$\mathcal{L}[\hat{\rho}_S] \coloneqq \frac{\mathrm{d}\hat{\rho}_S}{\mathrm{d}t} = -i[\hat{H}_S, \hat{\rho}_S] + \frac{1}{2}\sum_{j,k>0} \kappa_{jk}\left(2\hat{F}_{jk}\,\hat{\rho}_S\,\hat{F}_{jk}{}^{\dagger} - \left\{\hat{F}_{jk}{}^{\dagger}\hat{F}_{jk}, \hat{\rho}_S\right\}\right)$$

Spectrum of the GKSL Superoperator

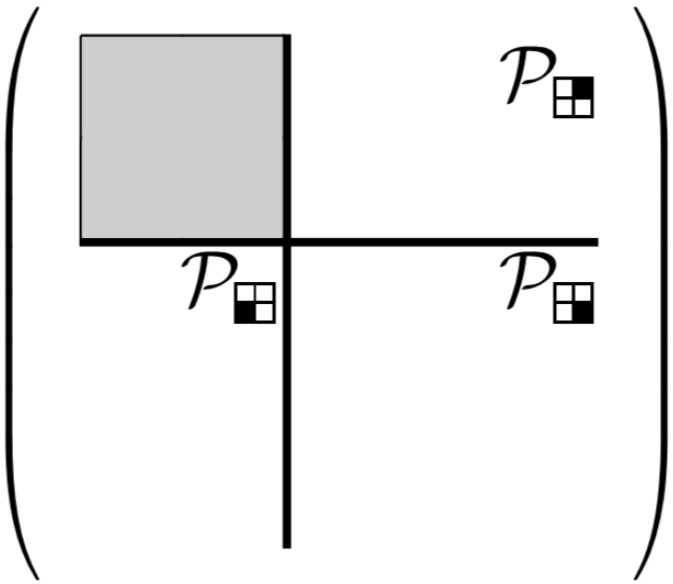
- Formal solution to GKSL equation: $\hat{\rho}_S(t) = e^{t\mathcal{L}}\,\hat{\rho}_S(0)$.
  - Examine spectral decomposition of $\mathcal{L}$ via Choi-Jamiołkowski isomorphism: mapping of operators $\mathbb{F}^{n\times m}$ to vector space $\mathbb{F}^{nm}$, employing Hilbert-Schmidt inner product.
    - $|a\rangle\langle b| \mapsto |a\rangle \otimes |b\rangle$; $\hat{A} \mapsto |\hat{A}\rangle\!\rangle$. Inner product $\langle\!\langle \hat{A}|\hat{B}\rangle\!\rangle = \mathrm{Tr}\{\hat{A}^\dagger \hat{B}\}$.
- Steady state solutions: defined as $|\hat{\rho}_{SS}\rangle\!\rangle \coloneqq \lim_{t\to\infty} e^{t\mathcal{L}}\,|\hat{\rho}_S(0)\rangle\!\rangle$.
- System with *multiple* steady states: assuming $\mathcal{L}$ is unitarily diagonalizable, have:

$$|\hat{\rho}_S(t)\rangle\!\rangle = e^{t\mathcal{L}}\,|\hat{\rho}_S(0)\rangle\!\rangle = \sum_a e^{t\lambda_a}\,|\hat{x}_a\rangle\!\rangle\langle\!\langle \hat{y}_a|\hat{\rho}_S(0)\rangle\!\rangle$$

  - $|\hat{x}_a\rangle\!\rangle$, $\langle\!\langle \hat{y}_a|$, λ_a are right eigenstates, left eigenstates, and eigenvalues of $\mathcal{L}$.
  - $\Re\{\lambda_a\} < 0$: damped states. $\Re\{\lambda_a\} = 0$: steady states. ($\Re\{\lambda_a\} > 0$ blows up.)

Four-Corners Decomposition

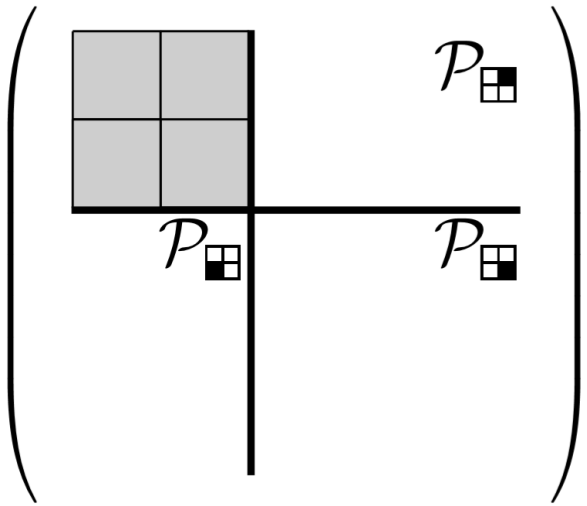
- Multiple steady states form[8] *asymptotic subspace* $\mathsf{As}(\mathcal{H})$ of nonzero steady states.
  - Right eigenvec. of $\mathcal{L}$ with pure imaginary eigenval.
    - *Corresponding* left eigenvec are system conserved currents.
    - All initial states have components along these states.
  - $\mathsf{As}(\mathcal{H})$ can have further nontrivial dynamics.
- *Four-corners decomposition*: decomposition of $\mathcal{L}$ in terms of $\mathsf{As}(\mathcal{H})$ and conserved currents.

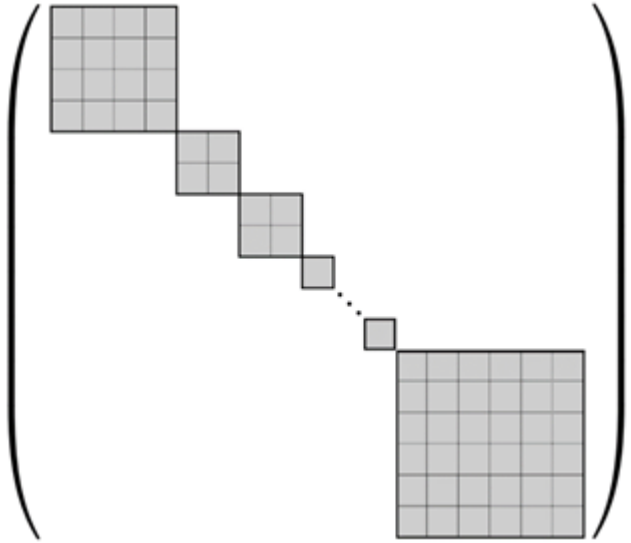


[8] – V. Albert, arXiv:1802.00010 (Ph.D. thesis, Yale). Image is from this.

Example Types of Asymptotic Subspaces

- Decoherent subspace: decoherence between steady states in dynamics purely within $\mathsf{As}(\mathcal{H})$.
  - Decoherence-free subspace (DFS): no further decoherence.
- Noiseless subsystem (NS): $\mathsf{As}(\mathcal{H})$ is tensor product of DFS subspace and subspace spanned by different steady state.
- von Neumann algebra: $\mathsf{As}(\mathcal{H})$ is a direct sum of DFS or NS blocks.
  - von Neumann algebra: C*-algebra which also includes all $\hat{A}$ that satisfy $\langle\phi|\hat{A}_n|\psi\rangle \to \langle\phi|\hat{A}|\psi\rangle$ for $|\phi\rangle, |\psi\rangle \in \mathcal{H}$ and $\{\hat{A}_n\} \in \mathsf{L}_1(\mathcal{H})$.

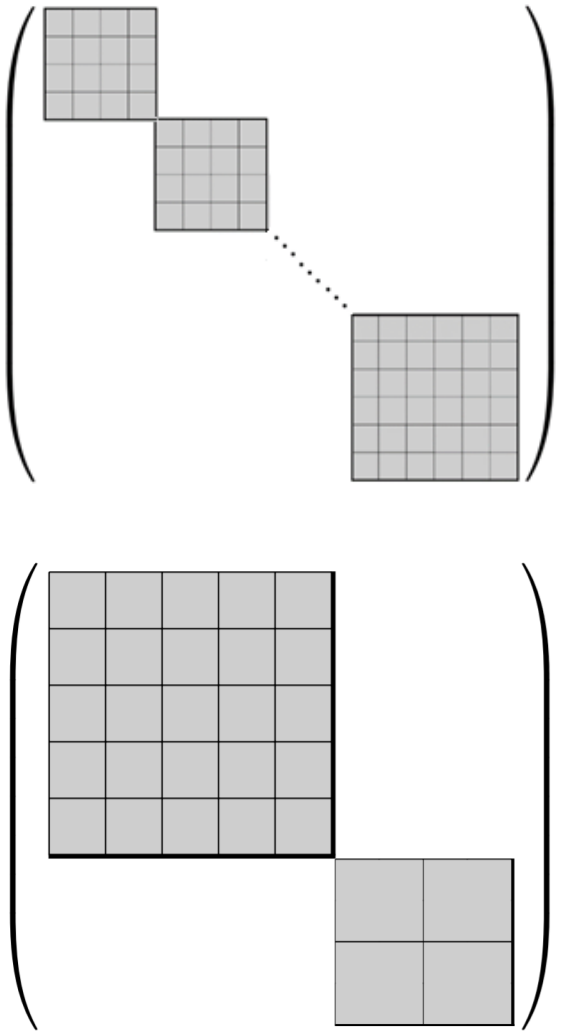




Images from (or modified from) V. Albert, arXiv:1802.00010 (Ph.D. thesis, Yale).

Generalized Reversible Computing and Four Corners

- Classical reversible computing: surjective map from physical to computational states, equivalence classes.
  - All states *within* a class must have same noncomputational entropy: related by a unitary transformation.
  - We permit intra-class coherences: each class can be a DFS.
  - Each class in a given computational scheme must have same computational entropy: each DFS block has same dimension.
  - ∴ quantum channels performing classical RC can be represented by the von Neumann algebra.
- Quantum GRC: permit inter-class coherences.



Images modified from V. Albert, arXiv:1802.00010 (Ph.D. thesis, Yale).

Berry Curvature and the Quantum Geometric Tensor

- Distance in parameter space:

$$\mathrm{d}s^2 = \|\psi(\lambda + \mathrm{d}\lambda) - \psi(\lambda)\|^2 = \langle \partial_\mu \psi | \partial_\nu \psi \rangle \, \mathrm{d}\lambda^\mu \, \mathrm{d}\lambda^\nu = \left(\gamma_{\mu\nu} + i\sigma_{\mu\nu}\right) \mathrm{d}\lambda^\mu \, \mathrm{d}\lambda^\nu$$

  - $\sigma_{\mu\nu}$: Berry curvature. $\sigma_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
  - $g_{\mu\nu} = \gamma_{\mu\nu} - A_\mu \wedge A_\nu$: quantum geometric tensor. Metric on the manifold of states.

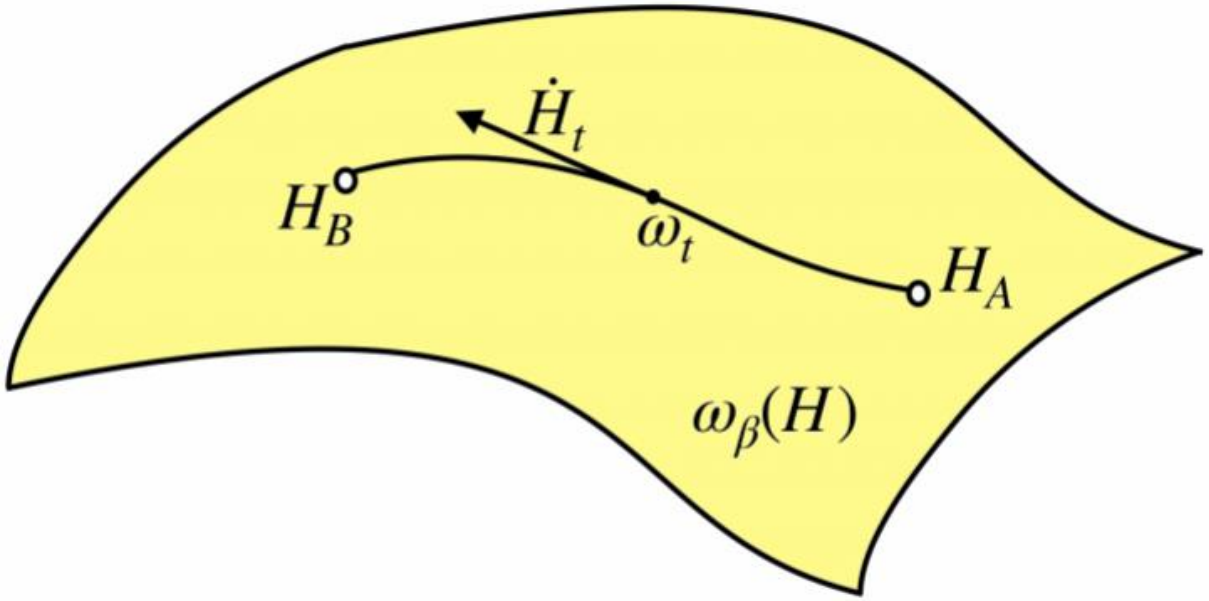
- Interest is in quantum geometric tensor induced by GKSL dynamics.
  - Berry connection $\mathcal{A}_{\alpha,\mu\nu} \coloneqq \langle\langle \hat{y}_\mu | \partial_\alpha | \hat{x}_\nu \rangle\rangle$.
  - Induced QGT for vN algebra: $\mathcal{Q}_{\alpha\beta,\mu\nu} = \partial_\alpha \mathcal{A}_{\beta,\mu\nu} + \mathcal{A}_{\alpha,\mu\nu} \wedge \mathcal{A}_{\beta,\mu\nu} - \langle\langle \hat{\rho}_\mu | \partial_\alpha \partial_\beta | \hat{\rho}_\nu \rangle\rangle$.

Dissipation-Delay Product

- Major quantity of interest for characterizing efficiency of reversible operations.
  - Goal: characterizing *general* (protocol-based, device-independent) efficiency properties of reversible operations.
- Dissipation-delay product: product of energy dissipation incurred by transition process and delay of process.
  - For perfectly unitary time evolution, dissipation is zero.
  - In principle, delay quantity is *not* subject to the quantum speed limit, but can be used as an initial value for a non-tight bound.
  - Want to quantify energy dissipation for reversible operation. Idea: use nonequilibrium quantum thermo.

Dissipation Length for Single Steady States

- *Thermodynamic* dissipation length: minimal dissipation amount for processes.
  - Construct similar manifold for set of all possible Gibbs states.
  - Geodesics: minimal dissipation process.
- Derived[9] for GKSL dynamics with a *single* steady state.



[9] – M. Scandi and M. Peranau-Lloblet, Quantum **3**, 197 (2019). Image from here.

Ballistic Quantum Computing

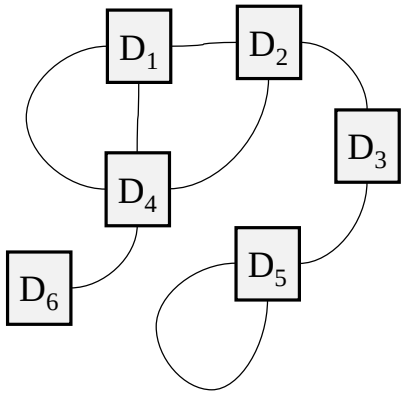
- Current: primitive quantum gate operations driven by external control (e.g. RF pulses). Several issues:
  - Control interfaces can be highly complex and can be a source of decoherence.
  - Circuit models aren't self-contained: difficult to examine physical properties (e.g. dissipation).
- Ballistic QC: qubits flying ballistically between devices in hardware circuit.
  - Gates as physical devices rather than operations; circuits as networks of connected devices rather than sequences of operations.
  - Devices carry local unitary transformations automatically via fixed, innate, designed-in $\widehat{H}$.

Asynchronous Ballistic Quantum Computing

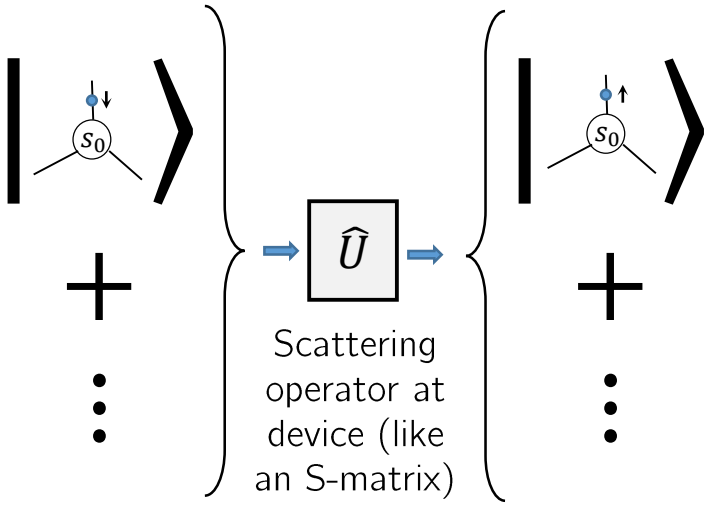
- Dynamics implemented by the qubit-device interaction in the ballistic model will generally be sensitive to their relative time of arrival.
  - Devices that require multiple interacting qubits to arrive simultaneously will be physically impossible to satisfy. Chaotic effects & Heisenberg both yield intrinsic timing uncertainty.
- *Asynchronous*: require qubits to arrive at times that are widely separated.
  - Need a model that can implement quantum gates even for somewhat noisy transmission lines, and independent of exact qubit arrival times.
  - Permits both stationary and flying qubits; interacting according to fixed $\widehat{U}$ at each device.
    - At each device, at most one flying qubit need interact at a time.

Constraints on ABQC Implementation Schemes

- Constraints on ABQC model, from both requirements on QC and requirements on asynchronous classical ballistic RC[10]. Constraints from QC:
  - Universal quantum gate set; robust or relatively noiseless channels propagating qubits; unitary invertibility.
- Additional constraints from asynchronous classical ballistic RC (ABRC)[11].



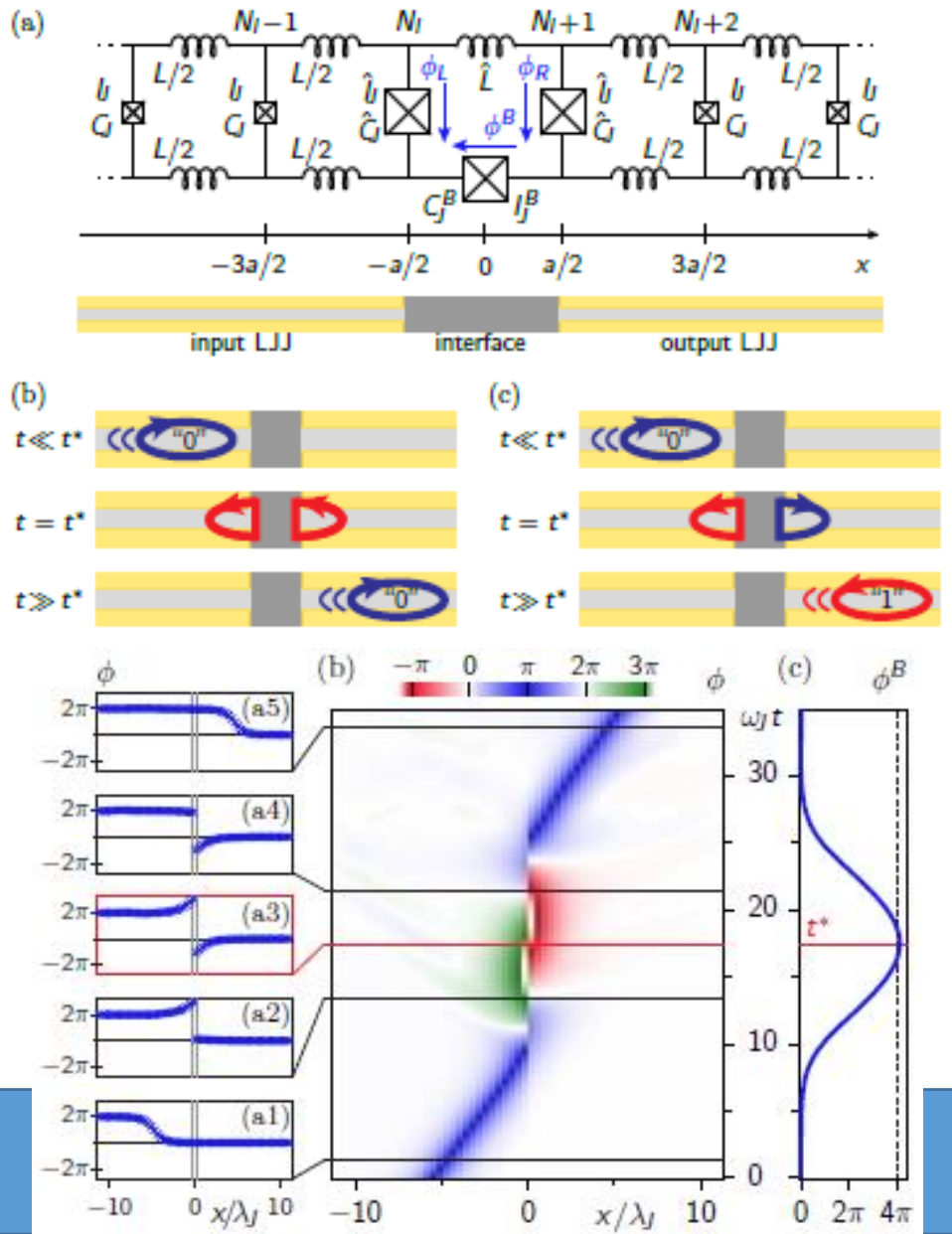
Network of hardware devices



[10] – M. P. Frank, in Proc. 2017 IEEE Int. Conf. Rebooting Comput., Institute of Electrical and Electronics Engineers, Incorporated, Red Hook (2017)
[11] – M. P. Frank, Asynchronous Ballistic Quantum Computing. Presentation: presented at the Quantum Coffee Hour, Sandia National Laboratories.

Physical Realizations of ABQC

- ABQC from ABRC:
  - General: devices implementing reversible trans. on finite sets $\rightarrow$ implementing unitaries on $\mathcal{H}$.
  - Extend simulation results[12] and ongoing experimental tests on RC via superconducting circuits by quantizing $\hat{\mathcal{L}}$? (Same as in circuit QED.)
- Ground-up model building might be simpler:
  - S-matrix construction of general ABQC model?
  - Use principles of ABQC to develop skyrmion-based model?



[12] – W. Wustmann and K. Osborn, arXiv:1711.04339. Images from here.

Conclusions and Next Steps

- Reversible computing: a new computing framework that can avoid dissipative costs involved in conventional computing.
- Current work: extending RC to a proper open system / nonequilibrium quantum thermodynamic footing. (Sandia Tracking #1068410)
  - Gives us fundamental quantum thermo results "for free"; e.g. thermodynamic length for multiple SSs.
  - Can give us a device-independent, protocol-based metric for efficiency of reversible operations!
- Next work: developing ABQC model.
  - Developing ground-up skyrmion based model.
  - Analogous to circuit QED, examining quantization conditions of classical ballistic RC model.

Nonequilibrium Landauer Limit

- Using RTQT, can get an explicit nonequilibrium Landauer bound[13].

$$\beta\langle Q\rangle \geq -\ln\left[\mathrm{Tr}_S\,\mathrm{Tr}_E\left\{\left(\hat{\rho}_S(0)\otimes \mathbb{1}_E\right)\widehat{U}_{SE}{}^{\dagger}\left(\mathbb{1}_S\otimes\hat{\rho}_{E,\beta}\right)\widehat{U}_{SE}\right\}\right]$$

- Dissipation bound depends on subsystem non-unitarity of $\widehat{U}_{SE}$.
  - Measure of non-unitarity[14] on E: $\mathcal{N}_E \coloneqq \left\|\sum_{jk}\lambda_k\langle s_k|\widehat{U}_{SE}|s_j\rangle\langle s_j|\widehat{U}_{SE}|s_k\rangle - \mathbb{1}_E\right\|$.
  - $\mathcal{N}_E = 0$: no lower bound on dissipation.
  - $\langle s_a|\widehat{U}_{SE}|s_b\rangle$: environment Kraus operators. Maps $\mathcal{D}(\mathcal{H}_E)$ to itself via operator algebra.

[13] – J. Goold, M. Paternostro, and K. Modi, Phys. Rev. Lett. **114**, 060602 (2015).
[14] – G. Guarnieri *et al.*, New J. Phys. **19**, 103038 (2017).

Berry Phase and Berry Connection

- Time evolution via adiabatic approx. $\widehat{H}(\lambda^\mu)$ parametrized by $\lambda^\mu(t) \in \mathbb{R}^n$:

$$|\psi(t)\rangle = e^{i\phi_n(t)} \exp\left\{-i \int_0^t \mathrm{d}\tau\, E_n(\lambda^\mu)\right\} |n(\lambda^\mu)\rangle$$

- Closed path in parameter space $\mathbb{R}^n$: returning to starting conditions. Permits nonzero gauge-invariant ϕ_n: Berry phase.
  - Degenerate eigenspaces: ϕ_n upgraded to $U(N)$ matrix. Non-Abelian Berry phase.
- Berry connection: gauge-dependent connection from Berry phase.

$$\phi_n(t) = \oint_C \mathrm{d}\lambda^\mu\, A_\mu\,; \qquad A_\mu = i\langle n(\lambda)|\partial_\mu|n(\lambda)\rangle$$

Practical Motivation for Reversible Computing

- Dennard scaling: power density of transistors remains constant as transistor size decreases.
- Moore's law: number of transistors in given integrated circuit area doubles (approximately) every 18 months.
  - Dennard: ended by 2008[15]. Moore: ending now[16].
- *Main* challenge: energy efficiency! Reversible computing: avoids major source of energy dissipation.
  - Almost all quantum computing is reversible, but classical reversible computing is the *lower bound* case on dissipation.

[15] – B. Deng *et al.*, ACM Trans. Arch. Code Opt. **15**, 8:1 (2018).
[16] – H. Khan, D. Hounshell, and E. Fuchs, Nature Electronics **1**, 14 (2018).