Fundamental Thermodynamic Limits of Classical Reversible Computing via Open Quantum Systems

Karpur Shukla [1, a], Victor V. Albert [2, 3], Michael P. Frank [4], and Jimmy Xu [1]

[1] *Laboratory for Emerging Technologies, Brown University, Providence, Rhode Island, 02912, U.S.A.*

[2] *Walter Burke Institute for Theoretical Physics &* [3] *Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California, 91125, U.S.A.*

[4] *Center for Computing Research, Sandia National Laboratories, Albuquerque, New Mexico, 87185, U.S.A.*

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1: Summary

In order to fully appreciate the fundamental limitations and advantages of classical reversible computing (CRC), and in order to understand how to practically implement CRC, it's imperative to first be able to express CRC operations from first principles. To easily exploit information theoretic aspects of quantum thermodynamics, the most natural framework to express CRC operations is in the language of quantum maps. However, although the principles of the Landauer cost in a quantum nonequilibrium framework have been known for some time [1], this has not yet translated into the development of actual models for CRC in terms of quantum maps. Furthermore, to examine the properties of CRC "in action", we want to be able to characterize the effects that environmental noise may play at fundamental levels.

Here, we identify as a **priority research challenge** the development of quantum-embedded models of CRC operations, and the use of this model to examine intrinsic (device-independent) thermodynamic properties of CRC operations. In particular, we seek to express logical operations and the effects of noise in terms of quantum maps. We can then use these expressions to characterise minimal entropy production for such operations, describe how to implement shortcuts-to-adiabaticity (STA), and most importantly develop practical physical models which can implement such operations. Our **proposed research program** is to apply the framework of *Lindbladians with multiple asymptotic states* [2–4] to developing models of CRC operations, which gives us a natural framework to express these operations in an open quantum system; and to use this framework to describe entropy production and thermodynamic dissipation for CRC operations.

2: Background

In order to exploit the quantum thermodynamic framework, we need to express classical reversible operations in terms of quantum maps. In real devices, several quantum states (possibly quite a large number of them) can map to the same computational state; in this light, we can follow models of reversible computing [5] where density matrices are surjective onto computational states. For a physically realistic system, we must also contend with environment-induced noise and systems where the initial conditions may result in dissipation into the environment. The effects of this system-environment interaction are captured in the theory of *open quantum systems*.

One standard framework for examining evolution in an open quantum system is the *Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) superoperator* $\mathcal{L}$ [6,7], which describes the time evolution of a density matrix under the Born-Markov approximation. For some initial state ρ_{in}, the *asymptotic states* ρ_∞ of $\mathcal{L}$ are the states that the system reaches in the infinite time limit: $\rho_\infty \coloneqq e^{t\mathcal{L}}[\rho_{\text{in}}]$. Although usually systems with only a single asymptotic state are considered, one of the authors (V. V. Albert, alongside B. Bradlyn, M. Fraas, and L. Jiang) recently developed [2–4] a framework of GKSL-evolving systems with *multiple* asymptotic states. Particularly notable about this framework is that the asymptotic states form a subspace As(H) in the overall dynamics, which can support unitary evolution and more complicated sub-subspace structures within it.

3: Applying Multiple Steady States to Classical Reversible Computing

Crucially for our purposes, As(H) provides a natural way to express CRC operations using the surjective framework in [5]. In the so-called *multi-block case* [2], the asymptotic subspace dynamics is represented by blocks (sub-subspaces), where density matrices within each block can have coherences internally (intra-block) but not between each other (inter-block). Taking inspiration from the expression of classical reversible operations via quantum maps in *closed* systems [8] and from the surjective framework, we can assign each block to a single computational state. Then, evolutions which preserve the multi-block structure represent non-computational evolutions which may mix density matrices within an equivalence class, while evolutions which don't conform to the multi-block structure represent computational operations. Examining the properties of these two different types of operations will help us understand the thermodynamic properties of classical reversible computing out of equilibrium and in open systems.

Several recent and important results [9-11] for systems with *single* asymptotic states are ripe for extension to multiple steady states, and allow us to characterise a variety of thermodynamic properties of classical reversible computing. The application of the multi-block framework to CRC operations, and the characterization of thermodynamic properties via this application, forms the research challenge we identify and the research program we propose. To give a non-exhaustive list of the specific questions in this program:

- One important question is the question of *minimal energy dissipation* for both of these types of processes. One technique to characterize this dissipation is by the technique of *thermodynamic length* [9] in open quantum systems under GKSL dynamics. Here, minimal dissipation of the time evolution of a Hamiltonian in an open quantum system is provided by examining the geodesic on the manifold of all possible thermal states. A complementary technique is to examine the *thermodynamic uncertainty relation*, which provides an uncertainty relation between precision and entropy production. Here, the uncertainty relation is characterized [10] by the covariance between the different currents in the asymptotic state. The extension of both of these results to systems with multiple asymptotic states (and specifically to the multi-block case) can allow us to characterize the dissipation properties of both the intra-block and inter-block operations.
- The *resource theory of quantum thermodynamics* provides a description of the thermal cost of single operations via the second laws of thermodynamics generated from the relative Rényi entropies. This has been recently used [11] to characterize the work cost of operations on qubits. Amongst other things, resource theory and thermo-majorization have demonstrated that an analogous concept to reversible computing can serve to permit thermal operations on qubits to have arbitrarily small work cost. One of the many benefits that extending this analysis to the multi-block case in open quantum systems can have is to help characterize the *one-shot* thermodynamic properties of both the intra-block and inter-block operations.
- Combination of these analyses with *shortcuts to adiabaticity* (STA) [12] and/or *quantum speed limits* (QSL) [13]. STA is a technique in which the final spectral distribution of an adiabatic process is achieved in a much faster time, by introducing an extra term in the overall Hamiltonian driving the dynamics of the system. Relatedly, the quantum speed limit gives us a description of the minimal length of time that a transformation between states may take for any given Hamiltonian. Exploring these questions in the context of CRC operations, and in particular providing a description of these processes within the asymptotic state subspace As(H), can shed some light onto quantities of interest for engineering classical reversible computing systems. One such example would be dissipation as a function of delay.

Notes and References

a – karpur_shukla@brown.edu

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