



# Information Flows in Reversible Computing Out of Equilibrium, with Applications to Models of Topological Quantum Computing

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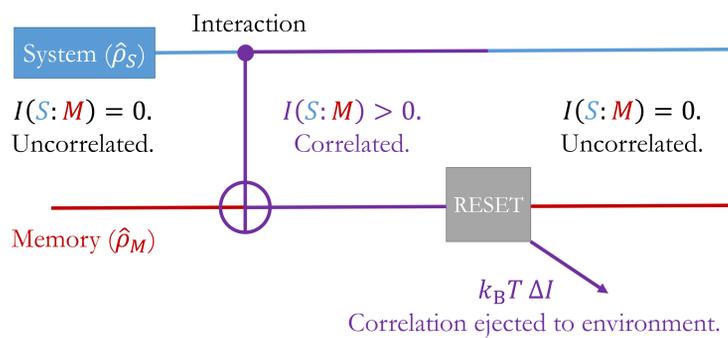


## I. Motivation

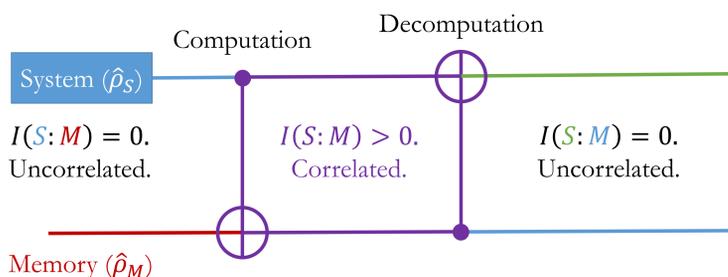
- **Landauer limit**<sup>[1]</sup>:  $k_B T \ln 2$  of free energy lost to heat in certain computational processes.
  - Fundamental lower bound of energy dissipated due to loss of information stored in bits to environment.
- **Reversible computing** paradigm developed as a means to improve energy efficiency of logical circuits.
  - Avoids Landauer cost: Avoids information ejection to environment.
- **Question 1:** *Can we reconcile apparently-differing viewpoints on Landauer erasure, and extend this to the resource theory of information in nonequilibrium thermodynamics?*
- **Question 2:** *Is there a quantum computational framework in which loss of physical information can be avoided via the underlying hardware throughout the computation process?*

## II. Information Flow and Correlations

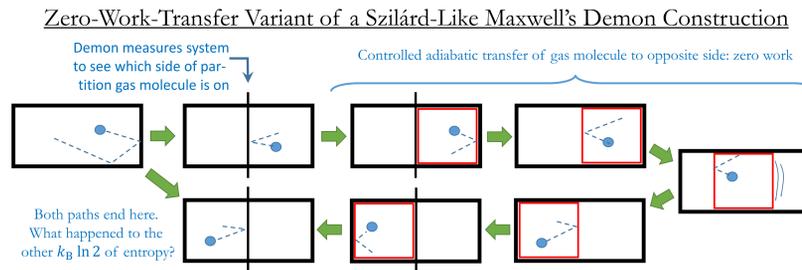
- Apparent disagreements<sup>[2-5]</sup> on the underlying nature of the Landauer bound.
  - Anderson<sup>[2]</sup>: comes from unconditional application of erasure protocols to reset state of a system coupled to environment.
  - Sagawa<sup>[3]</sup>: feedback control provides thermodynamic reversibility for “information engines” (e.g. Szilárd) overall.
- Which view is accurate? Both!
  - Landauer: Specifically focused on ejection of *correlated* bits from a controlled to an uncontrolled environment.



- Can now *define* a computation as any process setting up new correlations (new mutual information) on system.
  - Decomputation: *Reversibly* removing said correlations.
  - Decoherence = environment computations on a system.



- To demonstrate the role that tracking correlations plays in calculating the work cost of computation and decomputation, we consider a modified form of the Szilárd engine.

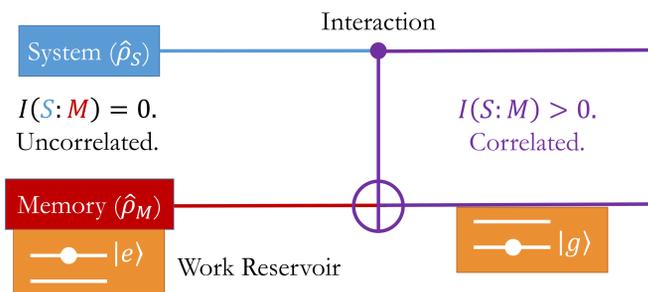


## III. Correlations Out of Equilibrium

- Information is a free energy resource<sup>[6-8]</sup> in thermo, giving rise to family of “ $\alpha$ -free energies”:

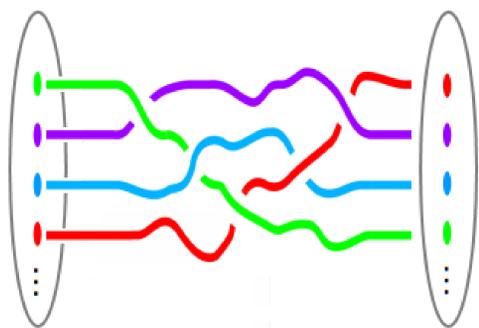
$$F_\alpha(\hat{\rho}) := -k_B T \ln \mathcal{Z} + k_B T S_\alpha(\hat{\rho} \| \hat{\rho}_G)$$

- $S_\alpha(\hat{\rho} \| \hat{\rho}_G)$  is  $\alpha$ -relative Rényi entropy between state  $\hat{\rho}$  and thermal (Gibbs) state  $\hat{\rho}_G$ .
- Müller<sup>[7]</sup>:  $\hat{\rho}_S \otimes \hat{\sigma}_M \otimes |e\rangle\langle e|_W \rightarrow \hat{\zeta}_{AM} \otimes |g\rangle\langle g|_W$  with  $\text{Tr}_M \hat{\zeta}_{AM} =: \hat{\tau}_S$  has free energy cost  $\Delta F$  arbitrarily close to free energy cost of pure  $\hat{\rho}_S \rightarrow \hat{\tau}_S$ .
- In other words, keeping correlations lets work cost be arbitrarily minimal (but nonzero): Avoids Landauer!

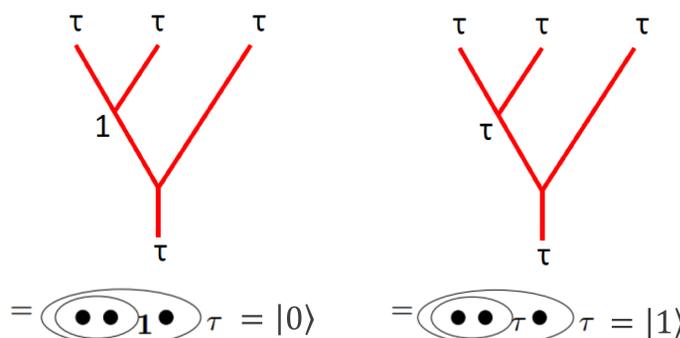


## IV. Topological Quantum Computation

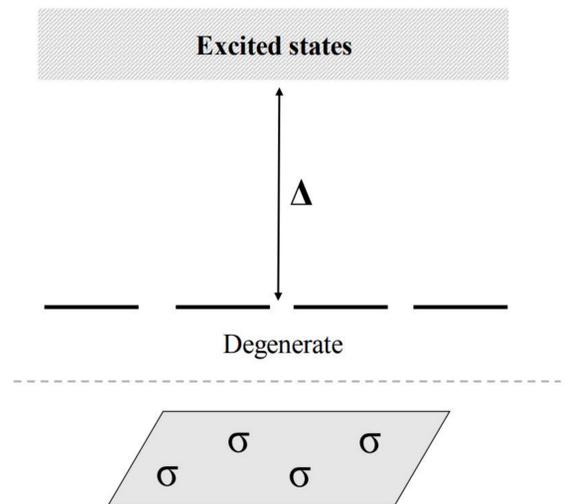
- Quantum computer, system described by topological effective theory. Realised by (2+1)D particles: anyons.
- Computations  $\in \text{SU}(2)$ : Braiding them around each other.



- Fusion: basis states given by possible fusion channels.



- Advantage: States are topologically protected from local perturbations.
  - Easiest way to enforce: Large energy gap between highly degenerate ground state and rest of the spectrum.



## V. Noneq. Holographic Entanglement

- Anyons arise<sup>[9]</sup> from (rational) conformal field theories.
  - Each distinct anyonic theory is characterised by a given Virasoro central charge /  $\text{SU}(2)$  level.
- Nonequilibrium aspects of 2D CFTs encoded in holographic entanglement entropy (AdS<sub>3</sub>/CFT<sub>2</sub> correspondence).
- By analogy to second laws, Bernamonti<sup>[10]</sup> *et al.* have calculated  $\alpha$ -RREs and work distribution functions for *quenched excited state*:
$$D_\alpha(\hat{\rho}_{\beta_E} \| \hat{\rho}_\beta) = \frac{\pi c L}{6\alpha - 6} \left( \frac{1}{\alpha\beta_E + (1-\alpha)\beta_E} - \frac{\alpha}{\beta_E} - \frac{1-\alpha}{\beta} \right)$$
  - $\beta_E$  is equilibrium temperature;  $L$  is thermal cylinder length.
- A similar technique is currently being investigated for *ground states* of adiabatically switched-on  $\text{SU}(2)_k$  non-Abelian Chern-Simons actions.
- Of interest: Viable in the dynamics of topological Fibonacci superconductors<sup>[11]</sup>?
  - Hope is to drive braiding operations using fluxons, in analogy to previous work on asynchronous ballistic reversible computing with conventional long Josephson junctions<sup>[12]</sup>.

## VII. Conclusions

- Tracking and managing correlations is essential for avoiding unnecessary entropy increase due to Landauer's principle.
- Work currently underway on examining nonequilibrium entropy flow for CFTs that support topological quantum computation.

## References

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