Information Flows in Reversible Computing Out of Equilibrium, with Applications to Models of Topological Quantum Computing

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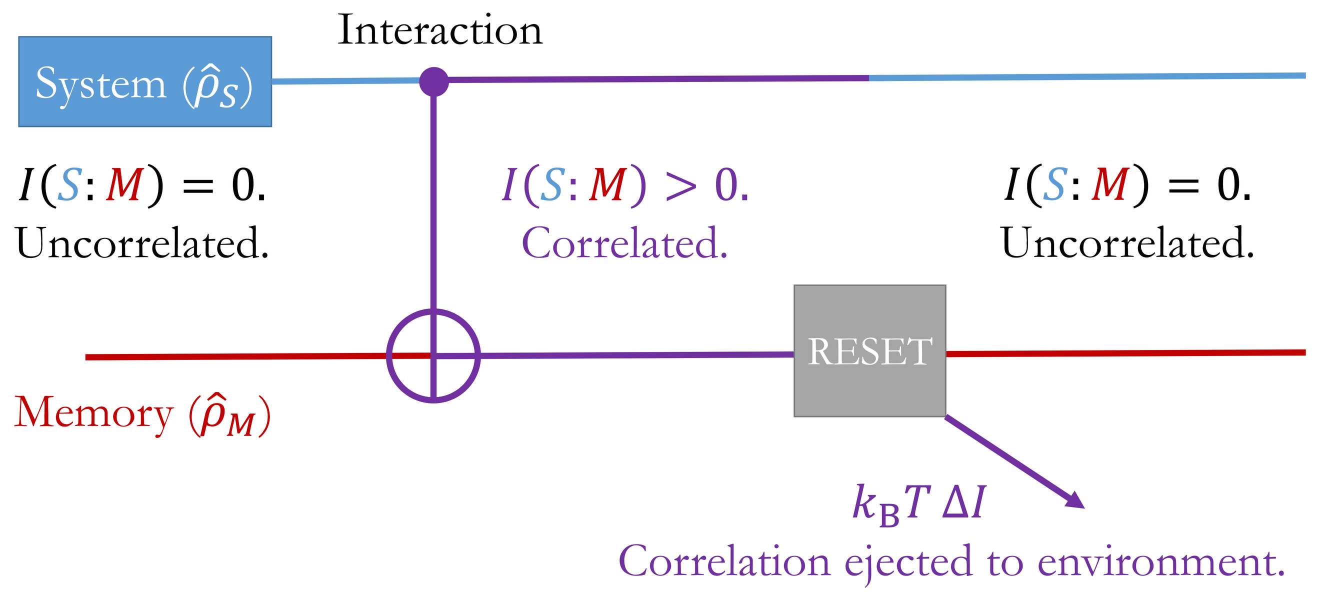
I. Motivation

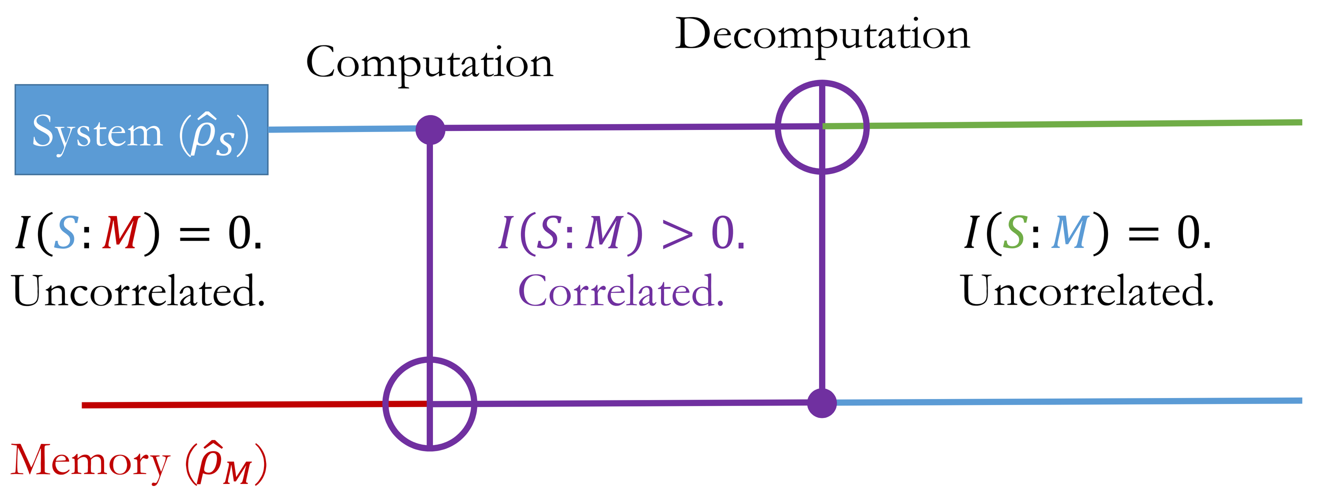
- *Landauer limit*[1]: $k_B T \ln 2$ of free energy lost to heat in certain computational processes.
  - Fundamental lower bound of energy dissipated due to loss of information stored in bits to environment.
- *Reversible computing* paradigm developed as a means to improve energy efficiency of logical circuits.
  - Avoids Landauer cost: Avoids information ejection to environment.
- **Question 1:** *Can we reconcile apparently-differing viewpoints on Landauer erasure, and extend this to the resource theory of information in nonequilibrium thermodynamics?*
- **Question 2:** *Is there a quantum computational framework in which loss of physical information can be avoided via the underlying hardware throughout the computation process?*

II. Information Flow and Correlations

- Apparent disagreements[2-5] on the underlying nature of the Landauer bound.
  - Anderson[2]: comes from unconditional application of erasure protocols to reset state of a system coupled to environment.
  - Sagawa[3]: feedback control provides thermodynamic reversibility for "information engines" (*e.g.* Szilárd) overall.
- Which view is accurate? Both!
  - Landauer: Specifically focused on ejection of *correlated* bits from a controlled to an uncontrolled environment.

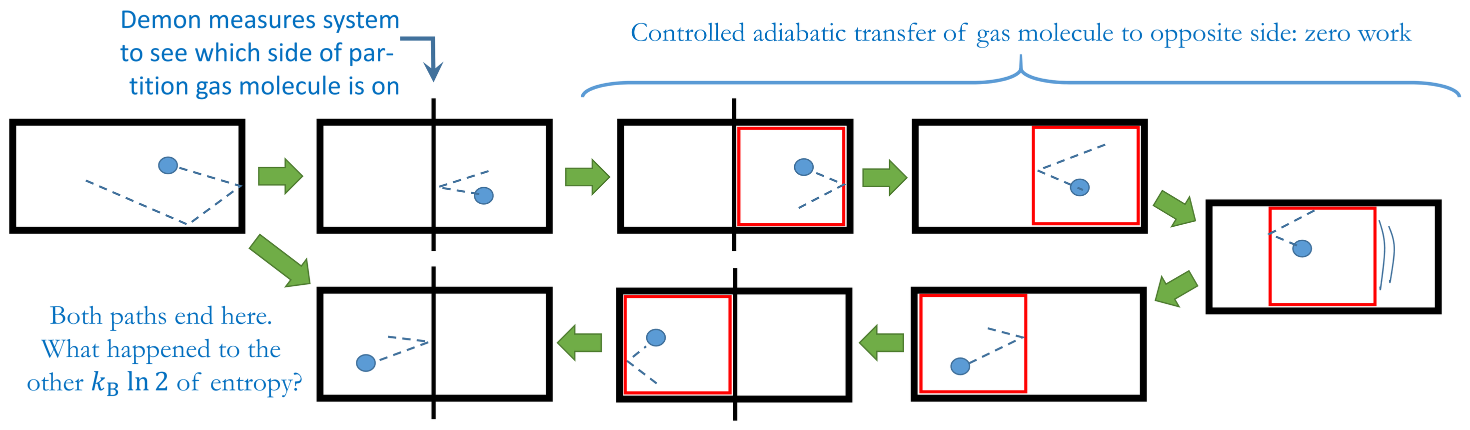


- Can now *define* a computation as any process setting up new correlations (new mutual information) on system.
  - Decomputation: *Reversibly* removing said correlations.
  - Decoherence = environment computations on a system.



  - To demonstrate the role that tracking correlations plays in calculating the work cost of computation and decomputation, we consider a modified form of the Szilárd engine.

Zero-Work-Transfer Variant of a Szilárd-Like Maxwell's Demon Construction

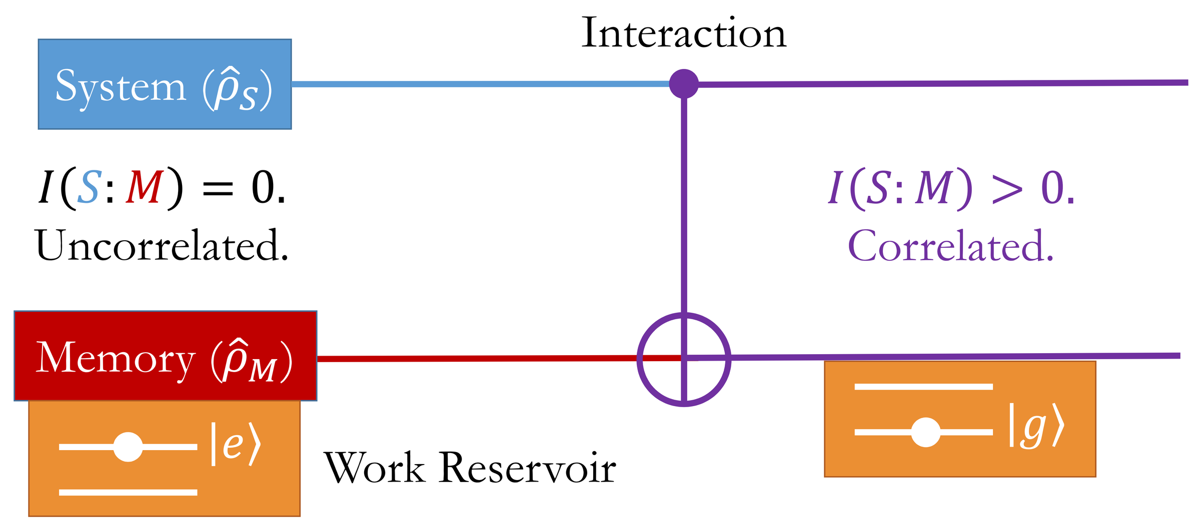


III. Correlations Out of Equilibrium

- Information is a free energy resource[6-8] in thermo, giving rise to family of "α-free energies":

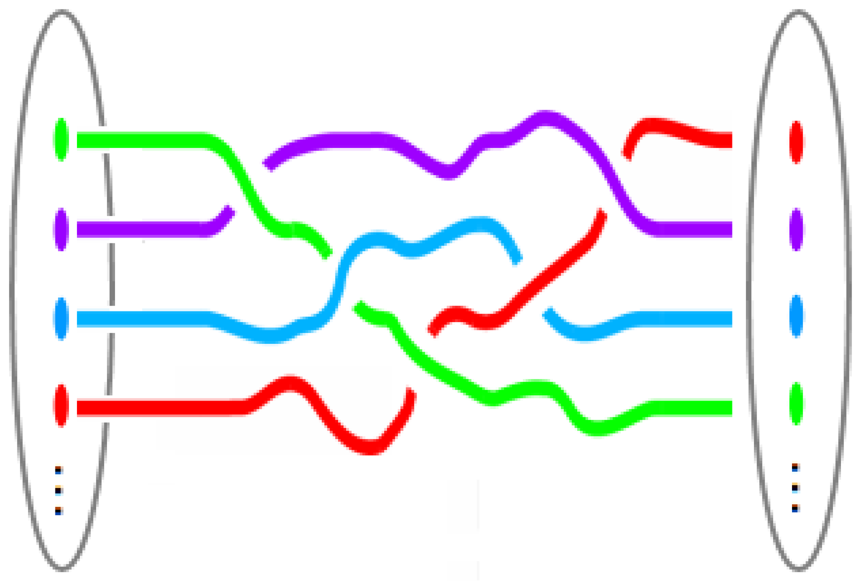
$$F_\alpha(\hat{\rho}) \coloneqq -k_B T \ln \mathcal{Z} + k_B T\, S_\alpha(\hat{\rho}\|\hat{\rho}_G)$$

  - $S_\alpha(\hat{\rho}\|\hat{\rho}_G)$ is α-relative Rényi entropy between state $\hat{\rho}$ and thermal (Gibbs) state $\hat{\rho}_G$.
- Müller[7]: $\hat{\rho}_S \otimes \hat{\sigma}_M \otimes |e\rangle\langle e|_W \to \hat{\varsigma}_{AM} \otimes |g\rangle\langle g|_W$ with $\mathrm{Tr}_M\, \hat{\varsigma}_{AM} \eqqcolon \hat{\tau}_S$ has free energy cost ΔF arbitrarily close to free energy cost of pure $\hat{\rho}_S \to \hat{\tau}_S$.
  - In other words, keeping correlations lets work cost be arbitrarily minimal (but nonzero): Avoids Landauer!

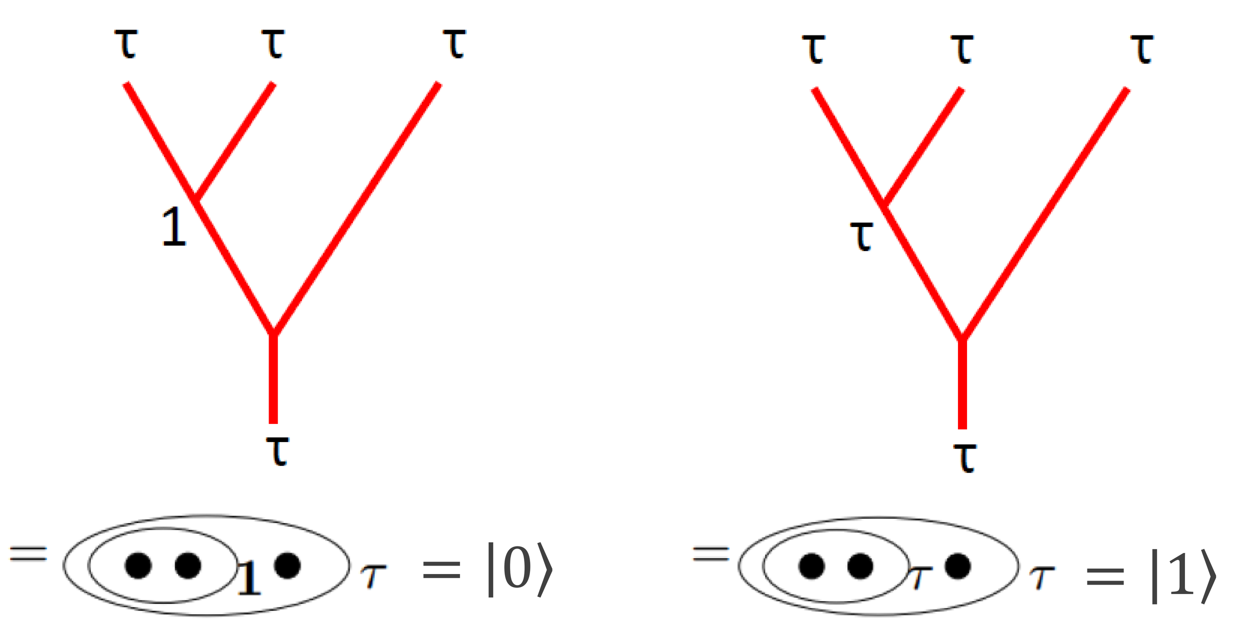


IV. Topological Quantum Computation

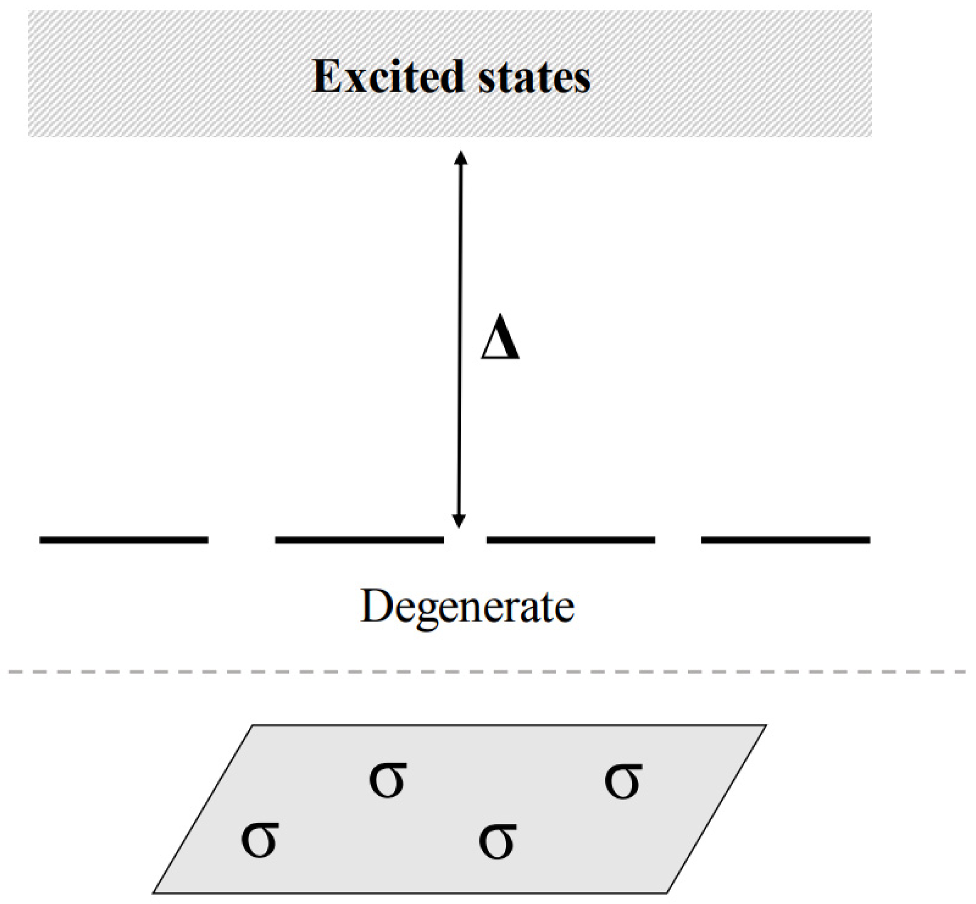
- Quantum computer, system described by topological effective theory. Realised by (2+1)D particles: anyons.
  - Computations ∈ SU(2): Braiding them around each other.



  - Fusion: basis states given by possible fusion channels.



- Advantage: States are topologically protected from local perturbations.
  - Easiest way to enforce: Large energy gap between highly degenerate ground state and rest of the spectrum.



V. Noneq. Holographic Entanglement

- Anyons arise[9] from (rational) conformal field theories.
  - Each distinct anyonic theory is characterised by a given Virasoro central charge / SU(2) level.
- Nonequilibrium aspects of 2D CFTs encoded in holographic entanglement entropy ($\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence).
- By analogy to second laws, Bernamonti[10] *et al.* have calculated α-RREs and work distribution functions for *quenched excited* state:

$$D_\alpha(\hat{\rho}_{\beta_E}\|\hat{\rho}_\beta) = \frac{\pi c L}{6\alpha - 6}\left(\frac{1}{\alpha\beta_E + (1-\alpha)\beta_E} - \frac{\alpha}{\beta_E} - \frac{1-\alpha}{\beta}\right)$$

  - β_E is equilibrium temperature; L is thermal cylinder length.
- A similar technique is currently being investigated for *ground* states of adiabatically switched-on $\mathrm{SU}(2)_k$ non-Abelian Chern-Simons actions.
- Of interest: Viable in the dynamics of topological Fibonacci superconductors[11]?
  - Hope is to drive braiding operations using fluxons, in analogy to previous work on asynchronous ballistic reversible computing with conventional long Josephson junctions[12].

VII. Conclusions

- Tracking and managing correlations is essential for avoiding unnecessary entropy increase due to Landauer's principle.
- Work currently underway on examining nonequilibrium entropy flow for CFTs that support topological quantum computation.

References

[1] – M. P. Frank, arXiv: 1901.10327
[2] – N. Anderson, Eur. Phys. J. B **91**, 156 (2018)
[3] – T. Sagawa, in *Energy Limits in Computation*, edited by C. S. Lent *et al.* (Springer Nature, Cham [Switzerland], 2019)
[4] – W. Porod, in *Energy Limits in Computation*, edited by C. S. Lent *et al.* (Springer Nature, Cham [Switzerland], 2019)
[5] – J. D. Norton, *Stud. Hist. Philos. Sci. B: Stud. Hist. Philos. Mod. Phys.* **42**, 184 (2011)
[6] – P. Strasberg *et al.*, Phys. Rev. X **7**, 021003 (2017)
[7] – M. Müller, Phys. Rev. X. **8**, 041051 (2018)
[8] – F. Brandão *et al.*, PNAS **112**, 3275 (2015)
[9] – C. Nayak *et al.*, Rev. Mod. Phys. **80**, 1083 (2008)
[10] – A. Bernamonti *et al.*, J. High Energy Phys. **2018**, 111 (2018)
[11] – Y. Hu and C. L. Kane, Phys. Rev. Lett. **120**, 066801 (2018)
[12] – M. P. Frank et al. (in preparation). IEEE Applied Superconductivity Conference 2018 poster available at: https://cfwebprod.sandia.gov/cfdocs/CompResearch/docs/ASC-poster.pdf

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