Ordinary Isotropic Peridynamic Models
Position Aware Viscoelastic (PAVE)
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John Mitchell
Center for Computing Research
Sandia National Laboratories, Albuquerque, New Mexico

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Ordinary peridynamic models: surface effects

*Position Aware* models correct for this

**Causes relate to material points near surface**

- Mathematical models assume all points are in the *bulk*
  - Points near surface are *missing bonds*
  - *Missing bonds* imply and induce incorrect material properties
  - In the bulk mathematical models are consistent

- Kinematic defects at the surface
Surface effects in ordinary peridynamic models

Tension test: ordinary isotropic elastic model (LPS)

The following related aspects contribute to mismatch.

- Geometric surface effects
- Nonlocal model kinematics
- Nonlocal model properties
- Discretization error
Review the practical issue/problem of surface effects

Introduce Position Aware models

Selecting/creating/evaluating influence functions (briefly)

Demonstration calculations

* Position Aware Linear Solid (PALS)
* Position Aware Viscoelastic (PAVE)
Maturation & extension of material models

- Bond-based
  - micro-brittle (PMB)
  - plasticity

- State-based
  - linear solid (LPS)
  - plastic
  - viscoelastic

- Position-aware
  - linear solid (PALS)
  - viscoelastic (PAVE)

generalized extended generalized

Surface

Missing bonds
Integral equation for internal force density $f$ of particle $x$

$$\rho(x)\ddot{u}(x,t) = f(x, u(x,t), t) + b(x,t)$$

$$f(x, u(x,t), t) = \int_H \left\{ T(Y)[x]\langle \xi \rangle - T(Y)[Q]\langle -\xi \rangle \right\} dV_Q$$
Ordinary material models

The vector force state $T$ is given as:

$$T(Y) = t(Y)M(Y)$$

where

$$M(Y) = \frac{Y}{|Y|}$$

Scalar force state $t(Y)$ defines ordinary material model. More later.
Kinematic peridynamic states: $e$, $\theta$, $\varepsilon$

Scalar extension state: $e$

$$e\langle \xi \rangle = |Y| - |\xi|$$

Dilatation: $\theta$

$$\theta = (\omega|\xi|) \cdot e$$
$$= \int_H \omega|\xi| e\langle \xi \rangle dV_Q$$

Deviatoric extension state: $\varepsilon$

$$\varepsilon = e - \frac{\theta|\xi|}{3}$$
Scalar force state obtained from elastic energy density functional

\[ W(\theta, \varepsilon) = \frac{\kappa \theta^2}{2} + \mu (\sigma \varepsilon) \cdot \varepsilon \]
Scalar force state obtained from elastic energy density functional

\[ W(\theta, \varepsilon) = \frac{\kappa \theta^2}{2} + \mu (\sigma \varepsilon) \cdot \varepsilon \]

Scalar force state

\[ t(Y) = p \omega x + 2\mu \sigma \varepsilon \]
Scalar force state obtained from elastic energy density functional

\[ W(\theta, \varepsilon) = \frac{\kappa \theta^2}{2} + \mu (\sigma \varepsilon) \bullet \varepsilon \]

Scalar force state

\[ t(Y) = p \omega x + 2\mu \sigma \varepsilon \]

Scalar force state with deviatoric *in-elastic* deformations \( \varepsilon^p \)

\[ t(Y) = p \omega x + 2\mu \sigma (\varepsilon - \varepsilon^p) \]

*elastic*
Scalar force state obtained from elastic energy density functional

\[ W(\theta, \varepsilon) = \frac{\kappa \theta^2}{2} + \mu_\infty (\sigma \varepsilon) \cdot \varepsilon + \sum_i \mu_i (\varepsilon - \varepsilon^i) \sigma \cdot (\varepsilon - \varepsilon^i) \]
Scalar force state obtained from elastic energy density functional

\[ W(\theta, \varepsilon) = \frac{\kappa \theta^2}{2} + \mu_\infty (\sigma \varepsilon) \cdot \varepsilon + \sum_i \mu_i (\varepsilon - \varepsilon^i) \sigma \cdot (\varepsilon - \varepsilon^i) \]

Scalar force state

\[ t(Y) = p \omega x + 2 \mu_\infty \sigma \varepsilon + 2 \sum_i \mu_i \sigma (\varepsilon - \varepsilon^i) \]
Scalar force state obtained from elastic energy density functional

\[ W(\theta, \varepsilon) = \frac{\kappa \theta^2}{2} + \mu_\infty (\overline{\sigma \varepsilon}) \cdot \varepsilon + \sum_i \mu_i (\varepsilon - \varepsilon^i) \overline{\sigma} \cdot (\varepsilon - \varepsilon^i) \]

Scalar force state

\[ t(Y) = p\omega x + 2\mu_\infty \overline{\sigma \varepsilon} + 2 \sum_i \mu_i \overline{\sigma} (\varepsilon - \varepsilon^i) \]

Governing equation for \( \varepsilon^i \)

\[ \dot{\varepsilon}^i + \frac{1}{\tau_i} \varepsilon^i = \varepsilon(t) \]
PALS (position aware linear solid) model

- \( \omega, \sigma \) are computed for each point in mesh
- Initial influence functions \( \omega^0, \sigma^0 \) given
- Select \( \omega, \sigma \) as best approximations to \( \omega^0, \sigma^0 \) subject to kinematic constraints: matching deformations

\[
\varepsilon^k \langle \xi \rangle = \frac{\xi \cdot H^k \xi}{|\xi|}
\]

\[
I(\omega, \lambda) = \frac{1}{2} (\omega - \omega^0) \cdot (\omega - \omega^0) - \sum_{k=1}^{K} \lambda^k \left[ (\omega_x) \cdot \varepsilon^k - \text{Tr} H^k \right]
\]

\[
N(\sigma, \tau) = \frac{1}{2} (\sigma - \sigma^0) \cdot (\sigma - \sigma^0) - \sum_{k=1}^{K} \tau^k \left[ (\sigma \varepsilon^k) \cdot \varepsilon^k - \gamma^k \right]
\]
Model problem: simple shear

PALS versus LPS: expectation *dilatation* $\theta = 0$

Simple shear

$$u = \tilde{\gamma} y; \quad v = 0; \quad w = 0; \quad \tilde{\gamma} = 1.0 \times 10^{-6}$$

**Dilatation**

LPS

PALS

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Model problem: simple shear

**PALS and PAVE**

\[ \tilde{\gamma} \]

Simple shear

- **Stored elastic energy density**

\[ W_L = \frac{1}{2} \mu \tilde{\gamma}^2 \]

\[ \mu = 25.90 \times 10^9 \]

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Crack Opening Displacement
Model Convergence

- Time (seconds)
- Crack opening displacement (m)

- num cells thickness = 6

- pave: λ = 0.99
- pave: λ = 0.50
- pave: λ = 0.01
- pals
- lps

MAG = 10^5

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Crack Opening Displacement
Mesh Convergence

Crack opening displacement (m) vs. Time (seconds)

- num cells thickness = (2, 6)
- pave: \( \lambda = 0.50 \)
- pals
- lps

MAG = 10^5

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Recover Young’s modulus $E$

Tensile test

PALS model: sharply reduces surface effects

PALS model: significant step toward making peridynamics accurate as a general-purpose simulation capability
Conclusions

▶ Reviewed the practical issue/problem of surface effects
▶ Introduced novel *Position Aware Linear Solid* model (PALS)
  * Addresses inaccuracies (LPS) due to missing bonds near surface
▶ Introduced novel *Position Aware Viscoelastic* model (PAVE)
▶ Demonstration calculations of *new* PAVE model
▶ Demonstration calculations show efficacy of PALS

THANK YOU

Questions?