Ordinary Isotropic Peridynamic Models Position Aware Linear Solid (PALS) SAND2014-15044PE

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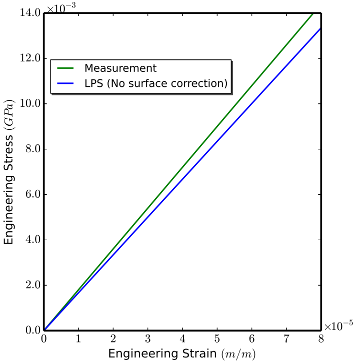
Outline

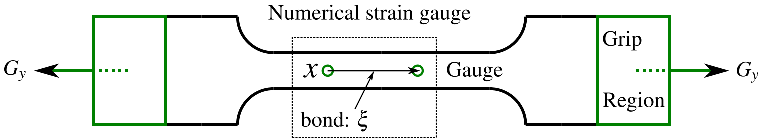
- Review the practical issue/problem of surface effects
- Introduce (PALS) & compare linear peridynamic solid (LPS)
- Selecting/creating/evaluating influence functions
- *Matching deformations*: dilatation, deviatoric
- Demonstration calculations verify efficacy of *PALS* model
- Summary, closing commments, and path forward

What is the *Dreaded Surface Effect*?
Example: Isotropic-Ordinary Model (LPS)



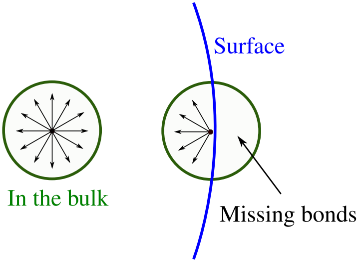
The following related aspects contribute to mismatch.

- Geometric surface effects
- Nonlocal model kinematics
- Nonlocal model properties
- Discretization error



Causes relate to material points near surface

- Mathematical models assume all points are in the *bulk*
  - Points near surface are *missing bonds*
  - *Missing bonds* imply and induce incorrect material properties
  - In the bulk mathematical models are consistent
- Isotropic ordinary materials have a *dilatation defect* at the surface



Isotropic ordinary elastic models
Compare *LPS* with *PALS*

Kinematics

$$\underline{e} = |\underline{\mathbf{Y}}| - |X| \qquad \underline{\varepsilon} = \underline{e} - \frac{\theta}{3}|X| \qquad \text{Bond: } \xi = x' - x = X\langle\xi\rangle$$

Linear peridynamic solid (LPS) model

$$W = \frac{1}{2}K\theta^2 + \frac{\alpha}{2}(\underline{\omega\varepsilon}) \bullet \underline{\varepsilon}, \qquad \theta = \frac{3}{m}(\underline{\omega}|X|) \bullet \underline{e}$$

$$m = \underline{\omega}|X| \bullet |X|, \qquad \alpha = \frac{15\mu}{m}$$

PALS model

$$W = \frac{1}{2}K\theta^2 + \mu(\underline{\sigma\varepsilon}) \bullet \underline{\varepsilon}, \qquad \theta = (\underline{\omega}|X|) \bullet \underline{e}$$

Compare *LPS* with *PALS*

Linear peridynamic solid model

- $\underline{\omega}$ is given and used for every point in mesh

PALS model

- $\underline{\omega}$, $\underline{\sigma}$ are computed for each point in mesh
- Initial influence functions $\underline{\omega}^0$, $\underline{\sigma}^0$ given
- Select $\underline{\omega}$, $\underline{\sigma}$ as best approximations to $\underline{\omega}^0$,$\underline{\sigma}^0$ subject to kinematic constraints: *matching deformations* $\underline{e}^k\langle\xi\rangle = \frac{\xi \cdot H^k \xi}{|\xi|}$

$$I(\underline{\omega}, \lambda) = \frac{1}{2}(\underline{\omega} - \underline{\omega}^0) \bullet (\underline{\omega} - \underline{\omega}^0) - \sum_{k=1}^{K} \lambda^k \left[(\underline{\omega x}) \bullet \underline{e}^k - \mathrm{Tr}\,\mathbf{H}^k \right]$$

$$N(\underline{\sigma}, \tau) = \frac{1}{2}(\underline{\sigma} - \underline{\sigma}^0) \bullet (\underline{\sigma} - \underline{\sigma}^0) - \sum_{k=1}^{K} \tau^k \left[(\underline{\sigma \varepsilon}^k) \bullet \underline{\varepsilon}^k - \gamma^k \right]$$

PALS: Selecting *dilatation* influence functions
Linear problem for Lagrange multipliers λ^k

Functional

$$I(\underline{\omega}, \lambda) = \frac{1}{2}(\underline{\omega} - \underline{\omega}^0) \bullet (\underline{\omega} - \underline{\omega}^0) - \sum_{k=1}^{K} \lambda^k \left[(\underline{\omega x}) \bullet \underline{e}^k - \mathrm{Tr}\, \mathbf{H}^k \right]$$

Variation

$$\delta I = \nabla I \bullet \delta \underline{\omega} + \sum_{k=1}^{K} \frac{\partial I}{\partial \lambda_k} \, \delta \lambda_k,$$

Substituting (2) into (1) gives linear problem for Lagrange multipliers

$$\frac{\partial I}{\partial \lambda^k} = 0 \quad \Longrightarrow \quad (\underline{\omega x}) \bullet \underline{e}^k = \mathrm{Tr}\, \mathbf{H}^k \qquad (1)$$

$$\nabla I = 0 \quad \Longrightarrow \quad \underline{\omega} = \underline{\omega}^0 + \sum_{k=1}^{K} \lambda^k \underline{x e}^k \qquad (2)$$

Matching deformations: *Sample Set*

Probe operator $e^k\langle\xi\rangle = \frac{\xi\cdot H^k\xi}{|\xi|}$

Dilatation

Let probe Δ be denoted by $\Delta = XX = YY = ZZ$

$$\underbrace{\begin{bmatrix} XX & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H^1} \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & YY & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H^2} \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ZZ \end{bmatrix}}_{H^3}$$

Let bond ξ components be denoted by $\{a, b, c\}$

$$e^1 = \frac{\Delta a^2}{|\xi|} \qquad e^2 = \frac{\Delta b^2}{|\xi|} \qquad e^3 = \frac{\Delta c^2}{|\xi|}$$

Matching deformations: *Sample Set*

Probe operator $e^k\langle\xi\rangle = \frac{\xi \cdot H^k \xi}{|\xi|}$

Deviatoric

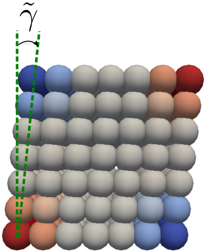
Let probe Δ be denoted by $\Delta = XY = XZ = YZ$

$$\underbrace{\begin{bmatrix} 0 & XY & 0 \\ XY & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{H^4} \underbrace{\begin{bmatrix} 0 & 0 & XZ \\ 0 & 0 & 0 \\ XZ & 0 & 0 \end{bmatrix}}_{H^5} \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & YZ \\ 0 & YZ & 0 \end{bmatrix}}_{H^6}$$

Let bond ξ components be denoted by $\{a, b, c\}$

$$e^4 = \frac{2ab\Delta}{|\xi|} \qquad e^5 = \frac{2ac\Delta}{|\xi|} \qquad e^6 = \frac{2bc\Delta}{|\xi|}$$

Model problem: simple shear
PALS versus *LPS*: expectation *dilatation* $\theta = 0$

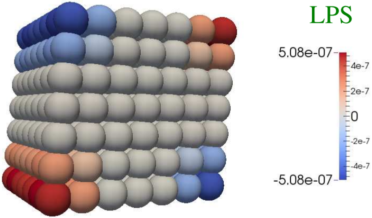


Simple shear

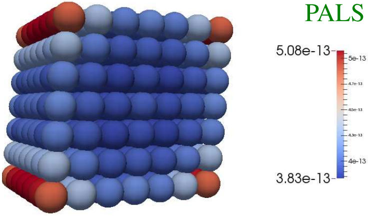
$$u = \tilde{\gamma} y; \quad v = 0; \quad w = 0; \quad \tilde{\gamma} = 1.0 \times 10^{-6}$$

Dilatation

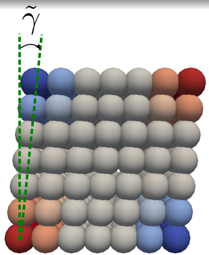
LPS



PALS



Model problem: simple shear
PALS versus *LPS*

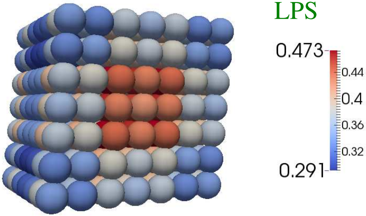


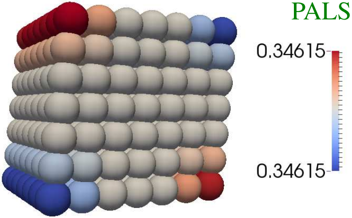
Simple shear

$u = \tilde{\gamma} y; \quad v = 0; \quad w = 0; \quad \tilde{\gamma} = 1.0 \times 10^{-6}$

$W_L = \frac{1}{2}\mu\tilde{\gamma}^2; \quad \mu = 6.923 \times 10^{11}; \quad W_L \approx .34615$

Stored elastic energy density

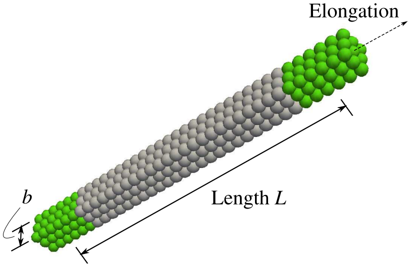


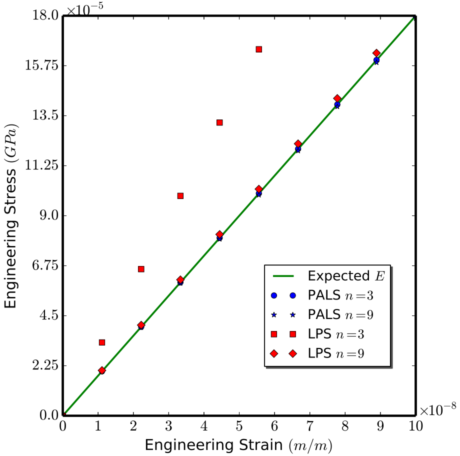


Demonstration calculation: Recover Young's Modulus E

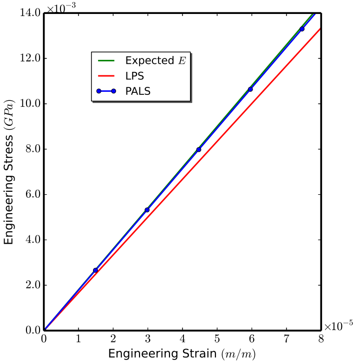
Influence functions: $\underline{\omega}^0 = \underline{\sigma}^0 = e^{\frac{-|\xi|^2}{\delta^2}}$

Property	Value
Edge length: b	0.5
Length: L	5.0
Num cells (along b): n	variable
Cell size: h	$h = b/n$
Horizon: δ	$3.1h$



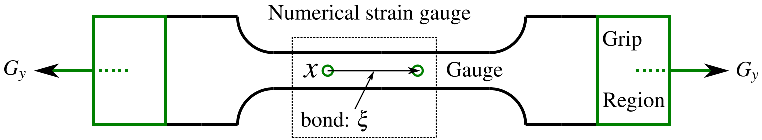


Re-visit tensile test using PALS and LPS
Influence functions: $\underline{\omega}^0 = \underline{\sigma}^0 = 1$



Results

- PALS sharply reduces error



Summary

- Reviewed the practical issue/problem of surface effects
- Introduced novel *Position Aware Linear Solid* model (PALS)
- PALS dilatation and energy density correct in pure shear
- Demonstration calculations show efficacy of PALS

THANK YOU

Questions?