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A Simple One Step Line Search For Use in Quasi-static Solid Mechanics Contact Calculations

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OUTLINE

A Simple One Step Line Search For Use in Quasi-static Solid Mechanics Contact Calculations

Preliminaries

- General class of problems under consideration (quasi-static equilibrium)
- Updated lagrangian solution strategy
- Linearization and the residual operator
- Linearization and our 1-step line search
- Contact enforcement: master/slave

Slip calculations and a simple line search (4 scenarios)

- Scenario I
- Scenario II
- Scenario III
- Scenario IV

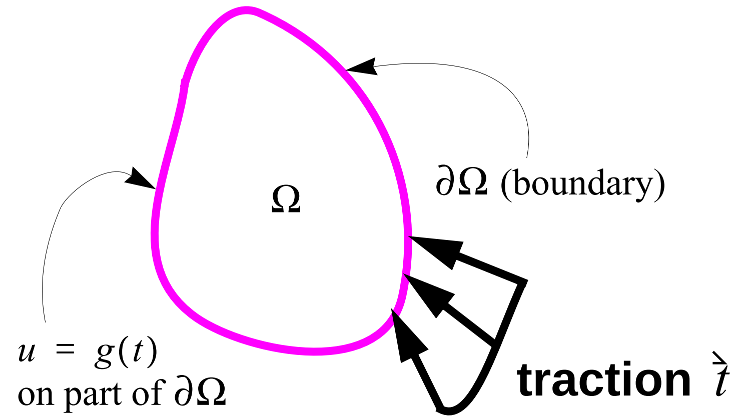
Adagio
quasi-static equilibrium

Strong form:

$$\nabla \cdot \bar{T} + \rho \vec{b} = 0$$

Weak form:

$$\int_{\Omega} \bar{T} : \delta \dot{\varepsilon} \, dV - \int_{\Omega} \rho \vec{b} \cdot \delta \vec{\phi} \, dV - \int_{\partial\Omega} \vec{t} \cdot \delta \vec{\phi} \, dV = 0$$



Ω = volume of body in current configuration

$\bar{T}$ = cauchy stress

$\vec{\phi}$ = virtual velocity field

$\delta \dot{\varepsilon} = \frac{1}{2}\left(\left(\frac{\partial}{\partial x}(\delta \vec{\phi})\right)^t + \left(\frac{\partial}{\partial x}(\delta \vec{\phi})\right)\right)$

quasi-statics: theory in which velocities are retained but time rate of velocities are neglected

Source of nonlinearities: Material, geometric, contact

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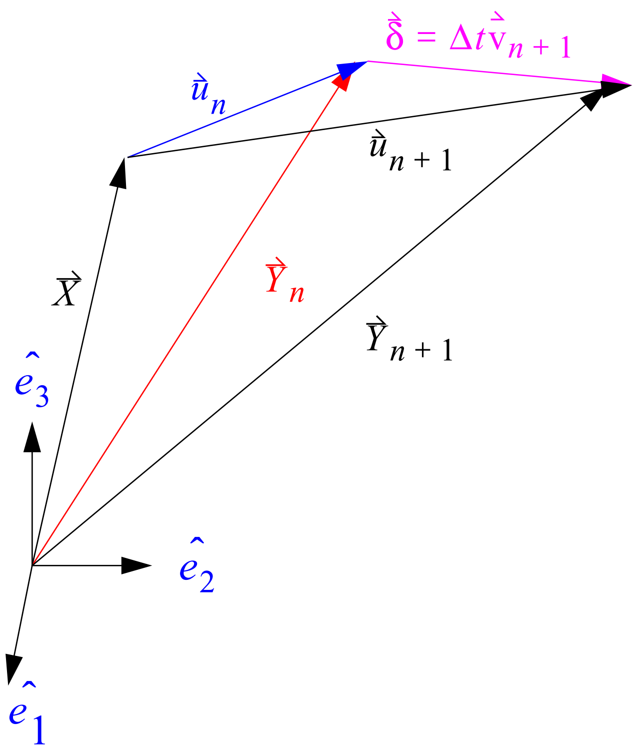
updated lagrangian solution strategy

Solution (velocity vector $\vec{v}_n$) is found at discrete points in time (t_n) by solving the nonlinear problem implied by the weak form



$\vec{X}$ **= material point at** $t = 0$

$\vec{u}_n$ **= displacement at step n**

$\vec{v}_n$ **= velocity at step n which leads to the displacement** $\vec{u}_n$

$\vec{\delta}$ **= incremental displacement**

$\vec{Y}_n$ **= current coordinates at step n**

$\Delta t = t_{n+1} - t_n$ **= time step size at**

Stresses and material state variables are updated at each time t_n

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Linearization and the directional derivative

Ignoring contact for the moment, we define the residual as:

$$R(\vec{u}_{n+1}, t_{n+1}) = \int_{\Omega} \bar{T} : \delta\dot{\varepsilon} \ dV - \int_{\Omega} \rho \vec{b} \cdot \delta\vec{\phi} \ dV - \int_{\partial\Omega} \vec{t} \cdot \delta\vec{\phi} \ dV \longrightarrow \text{force imbalance}$$

A taylor series expansion about the last equilibrium configuration is written as:

$$R(\vec{u}_n + \alpha\vec{\delta}) = R(\vec{u}_n) + \alpha\frac{d}{d\alpha}[R(\vec{u}_n + \alpha\vec{\delta})]\Big|_{\alpha = 0} + \frac{\alpha^2}{2}\frac{d^2}{d\alpha^2}[R(\vec{u}_n + \alpha\vec{\delta})]\Big|_{\alpha = 0} + HOT$$

By virtue of the equilibrium configuration at $\vec{u}_n$, the residual calculation is (to first order approximation) the directional derivative at $\vec{u}_n$ in the direction of the velocity field $\vec{v}_{n+1}$, where $\delta = \Delta t \vec{v}_{n+1}$.

Action of the residual operator yields the action of the tangent operator

$$R(\vec{u}_n + \alpha\vec{\delta}) \cong \alpha\frac{d}{d\alpha}[R(\vec{u}_n + \alpha\vec{\delta})]\Big|_{\alpha = 0} = \alpha R'(\vec{u}_n)\vec{\delta}$$

Adagio::line search and linearization

Our current line search calculation uses the following 1-step formula:

$$\alpha = -\frac{\vec{\delta}^{t} \cdot R(\widehat{u})}{\vec{\delta}^{t} \cdot [R(\widehat{u} + \vec{\delta}) - R(\widehat{u})]}$$

where $\widehat{u} = \vec{u}_n + \vec{\delta}_k$**, with** $\vec{\delta}_k$ **as the current iterate and** $\vec{\delta}$ **as the search direction**

Using the linearization concepts we have:

$$R(\widehat{u}) = R(u_n) + R'(u_n) \cdot \vec{\delta}_k \textbf{; and } R(\widehat{u} + \eta\vec{\delta}) = R(u_n) + R'(u_n) \cdot \vec{\delta}_k + \eta(R'(\widehat{u}) \cdot \vec{\delta})$$

Substituting into the 1-step formula above yields:

$$\alpha = -\frac{\vec{\delta}^{t} \cdot R'(u_n) \cdot \vec{\delta}_k}{\vec{\delta}^{t} \cdot R'(\widehat{u}) \cdot \vec{\delta}}$$

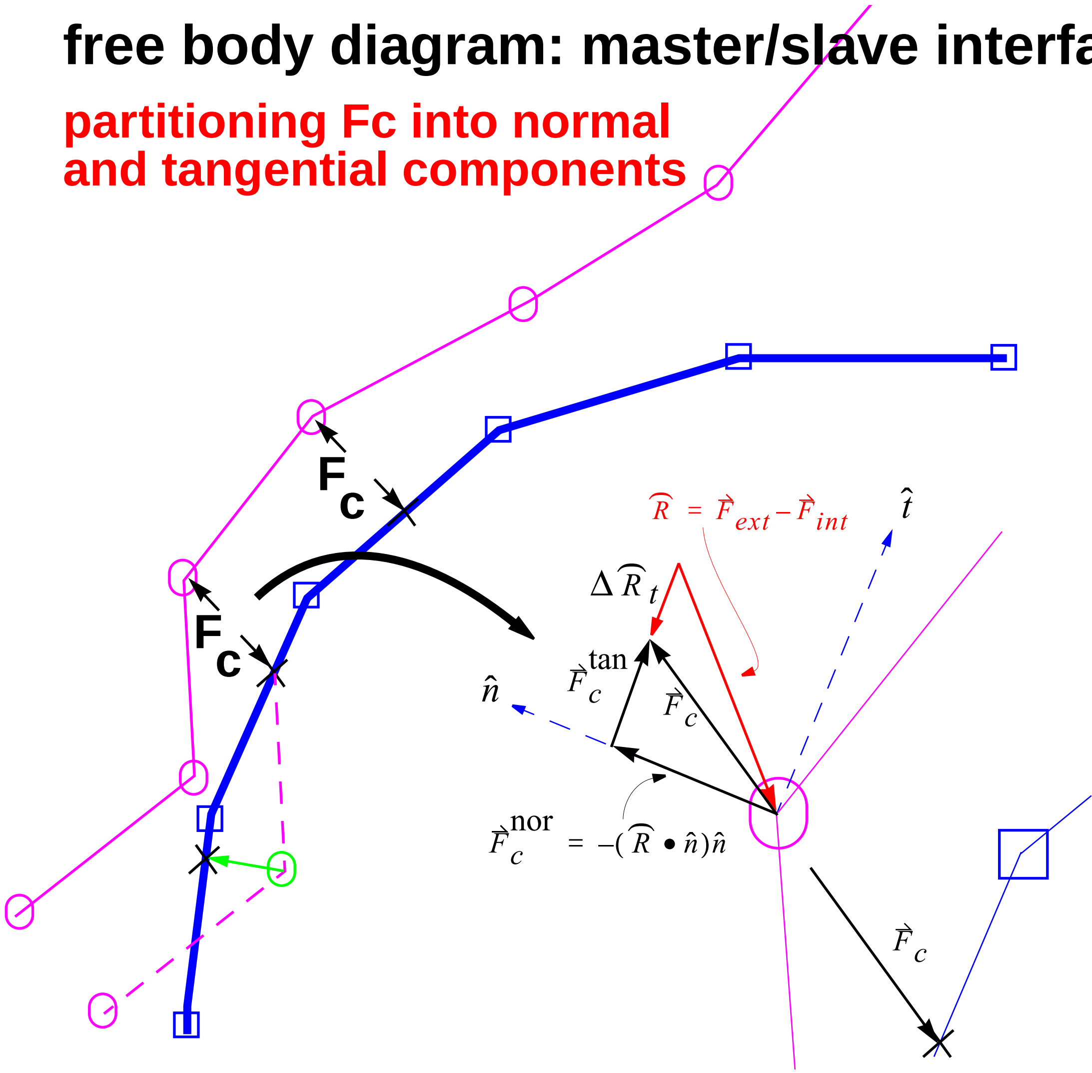
Notes

- positive-definite denominator
- projection of current force increment onto search direction in numerator

Adagio contact mechanics
Contact force calculation / resolution

free body diagram: master/slave interface

partitioning Fc into normal and tangential components



$\vec{R}$ - residual $= \vec{F}_{ext} - \vec{F}_{int} + \vec{F}_c$

$\vec{F}_c$ - contact force

$\hat{n}$ - unit normal to master surface

$\hat{t}$ - unit tangent to master surface

$\vec{F}_c^{\,tan}$ - frictional force

Line search and contact slip

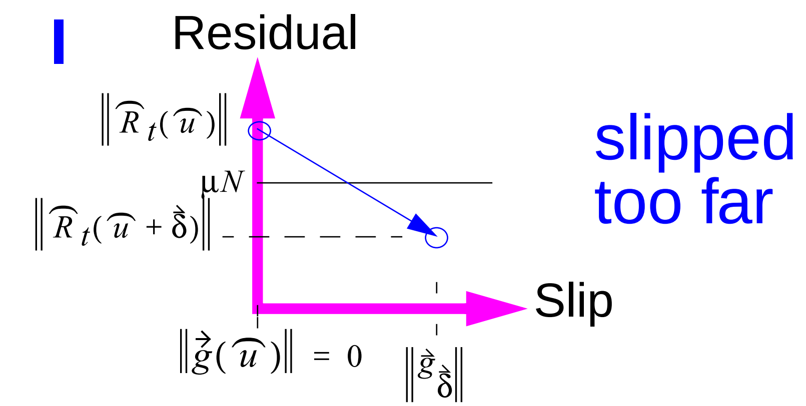
Direct application of line search formula:

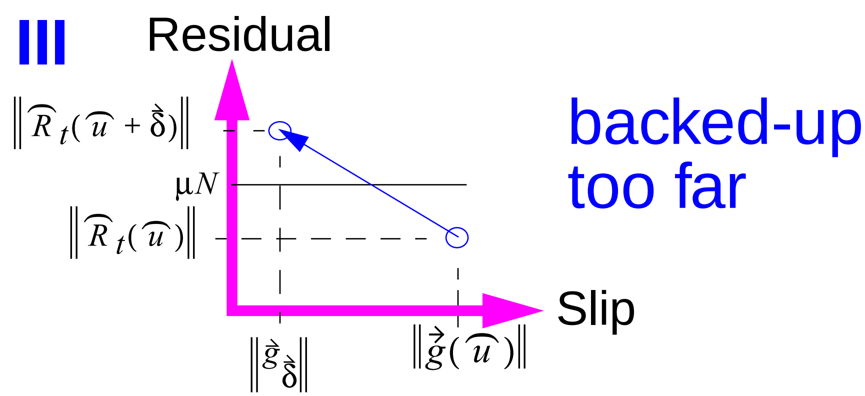
$$\alpha = -\frac{\vec{\delta}^{t} \cdot R(\widehat{u})}{\vec{\delta}^{t} \cdot [R(\widehat{u} + \vec{\delta}) - R(\widehat{u})]}$$

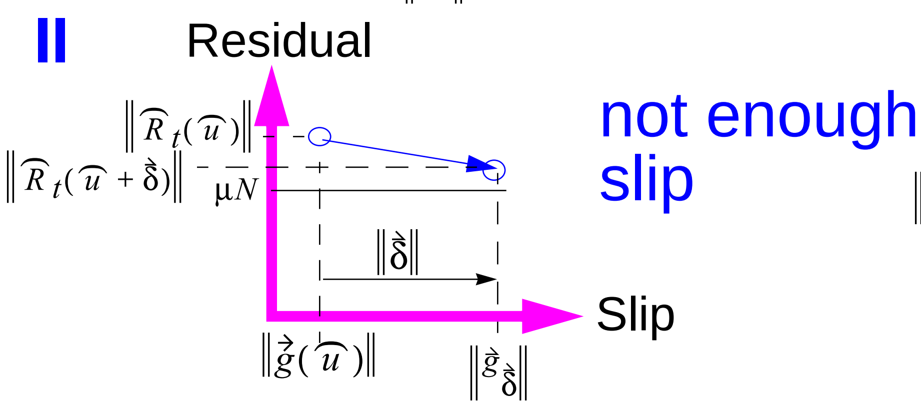
Issue: sticking/slipping conditions at $R(\widehat{u})$ and $R(\widehat{u} + \vec{\delta})$

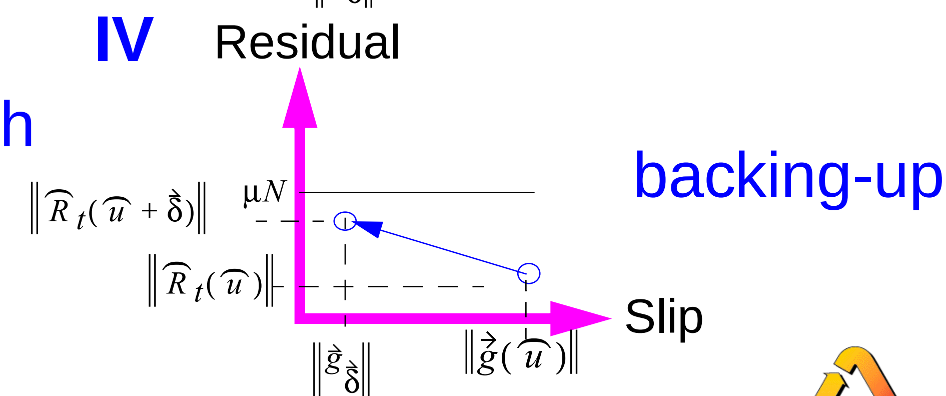
Define: $\widehat{R}(\vec{x}) = Fext(\vec{x}) - Fint(\vec{x})$; Residual imbalance less contact force

Schematic of scenarios:









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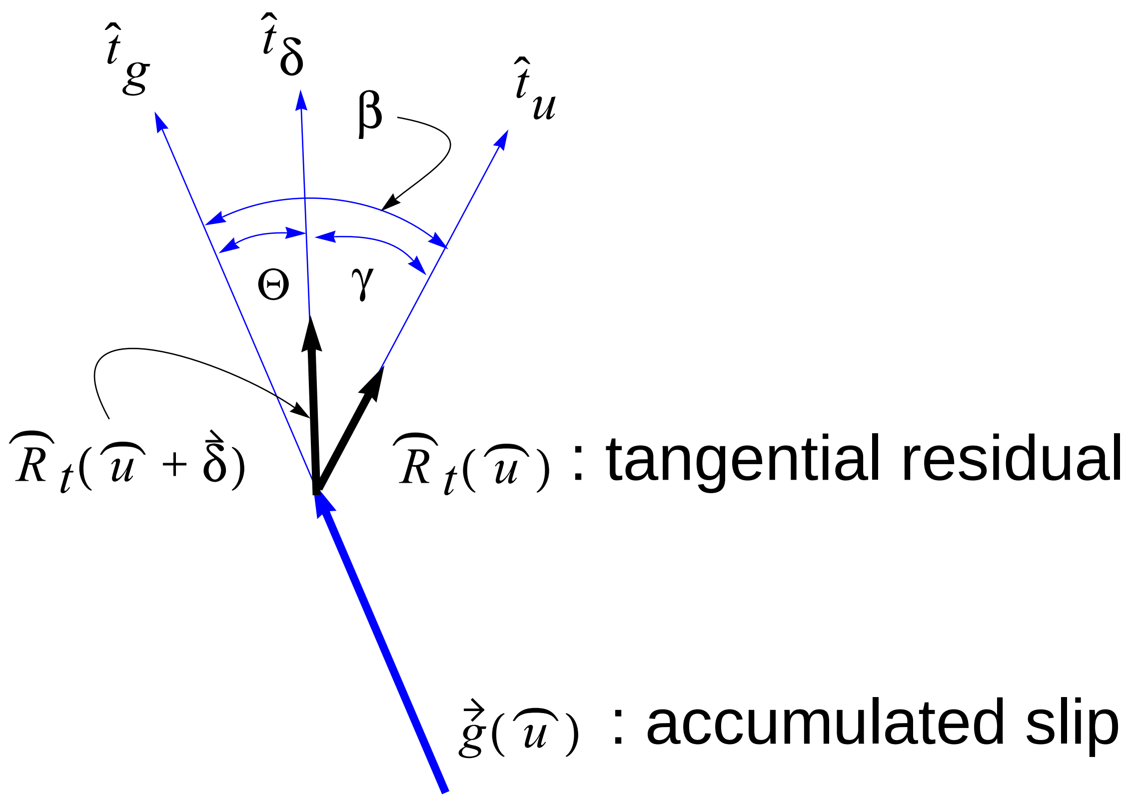
Line search and contact slip
Schematic of geometry



Line search and contact slip
Summary for scenario I

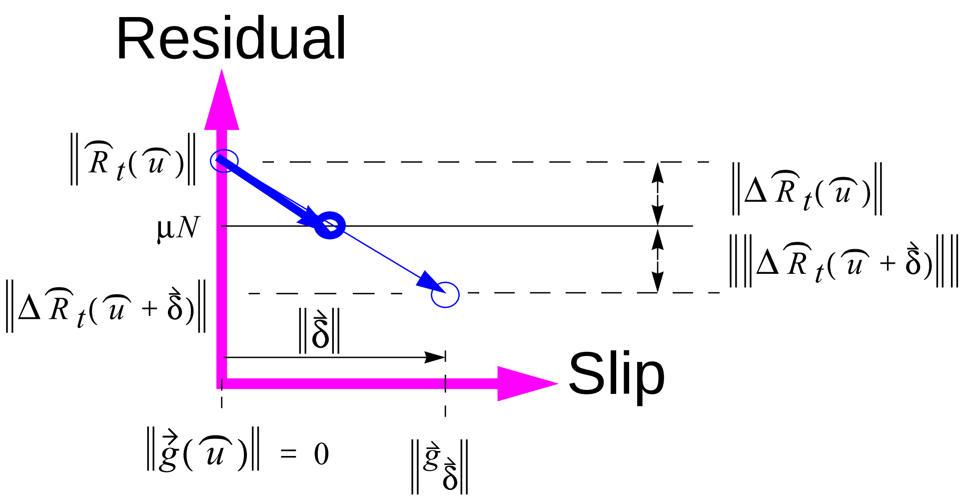
Consider the slip calculation where

$$\vec{\delta} = \|\vec{\delta}\| \, \hat{t}_u \,, \qquad \hat{t}_u = \frac{\widehat{R}_t(\widehat{u})}{\|\widehat{R}_t(\widehat{u})\|}$$

slipped too far

Then line search yields:

$$\alpha = \frac{\|\Delta \widehat{R}_t(\widehat{u})\|}{\|\Delta \widehat{R}_t(\widehat{u} + \vec{\delta})\| + \|\Delta \widehat{R}_t(\widehat{u})\|}$$



Line search and contact slip
Summary for scenario II

Use a search direction:

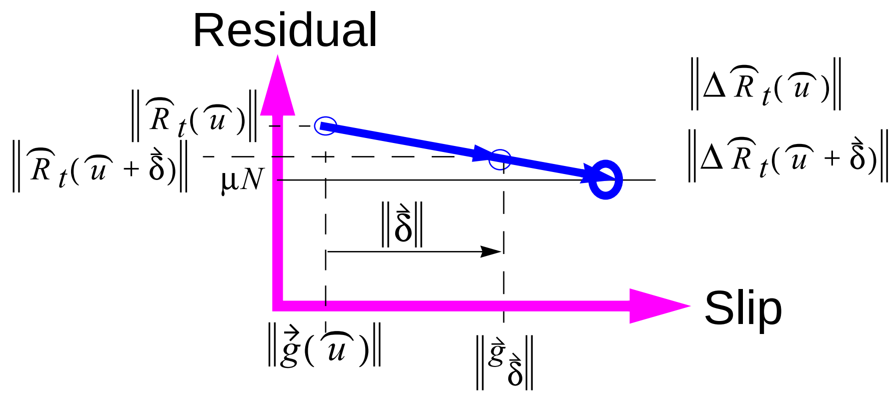
$$\vec{\delta} = \|\vec{\delta}\|\,\hat{t}_u,$$

where

$$\hat{t}_u = \frac{\widehat{R}_t(\widehat{u})}{\|\widehat{R}_t(\widehat{u})\|}, \qquad \hat{t}_\delta = \frac{\widehat{R}_t(\widehat{u}+\vec{\delta})}{\|\widehat{R}_t(\widehat{u}+\vec{\delta})\|},$$

$$\hat{t}_u \cdot \hat{t}_\delta = \|\hat{t}_u\|\,\|\hat{t}_\delta\|\cos\gamma$$

not enough slip



Then line search yields:

$$\alpha = \frac{\|\Delta\widehat{R}_t(\widehat{u})\|}{\|\Delta\widehat{R}_t(\widehat{u})\| - \|\Delta\widehat{R}_t((\widehat{u}+\vec{\delta}))\|\cos\gamma}$$

Line search and contact slip
Summary for scenario III

Use a search direction:

$$\vec{\delta} = -\|\vec{\delta}\|\,\hat{t}_g\ ,$$

where

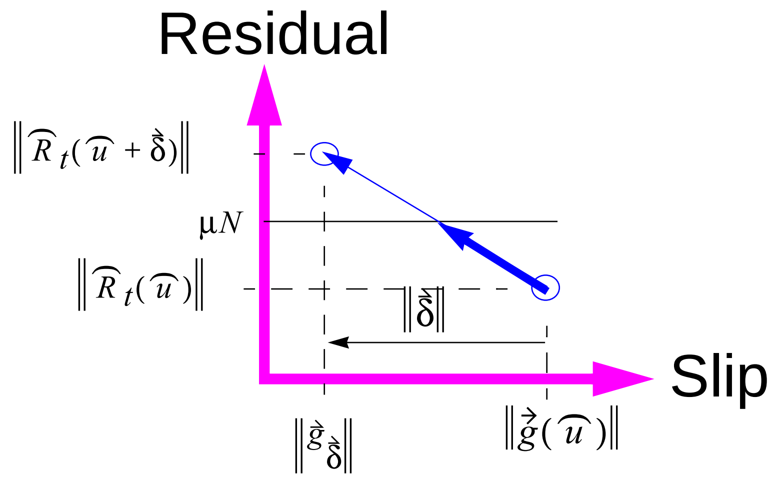
$$\hat{t}_u = \frac{\widehat{R}_t(\widehat{u})}{\|\widehat{R}_t(\widehat{u})\|}\ , \qquad \hat{t}_g = \frac{\vec{g}(\widehat{u})}{\|\vec{g}(\widehat{u})\|}\ ,$$

$$\hat{t}_u \cdot \hat{t}_g = \|\hat{t}_u\|\,\|\hat{t}_g\|\cos\beta\ , \quad \hat{t}_g \cdot \hat{t}_\delta = \|\hat{t}_g\|\,\|\hat{t}_\delta\|\cos\theta$$

Then line search yields:

$$\alpha = \frac{\|\Delta\widehat{R}_t(\widehat{u})\|\cos\beta}{\|\Delta\widehat{R}_t(\widehat{u}+\vec{\delta})\|\cos\theta + \|\Delta\widehat{R}_t(\widehat{u})\|\cos\beta}$$

backed-up too far



Line search and contact slip
Summary for scenario IV

Use a search direction:

$$\vec{\delta} = -\|\vec{\delta}\|\,\hat{t}_g\,,$$

where

$$\hat{t}_u = \frac{\widehat{R}_t(\widehat{u})}{\|\widehat{R}_t(\widehat{u})\|}\,, \qquad \hat{t}_g = \frac{\vec{g}(\widehat{u})}{\|\vec{g}(\widehat{u})\|}\,,$$

$$\hat{t}_u \cdot \hat{t}_g = \|\hat{t}_u\|\,\|\hat{t}_g\|\cos\beta\,, \quad \hat{t}_g \cdot \hat{t}_\delta = \|\hat{t}_g\|\,\|\hat{t}_\delta\|\cos\theta$$

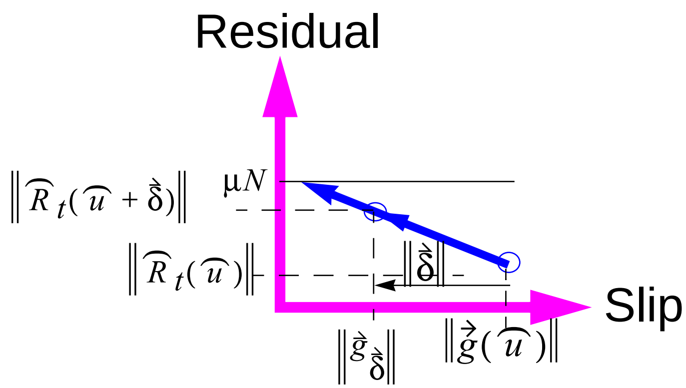
Then line search yields:

$$\alpha = \frac{\|\Delta\widehat{R}_t(\widehat{u})\|\cos\beta}{\|\Delta\widehat{R}_t(\widehat{u})\|\cos\beta - \|\Delta\widehat{R}_t(\widehat{u}+\vec{\delta})\|\cos\theta}$$

NOTE: Slope on residual

if $\alpha\|\vec{\delta}\| > \|\vec{g}(\widehat{u})\|$, then $\alpha = \dfrac{\|\vec{g}(\widehat{u})\|}{\|\vec{g}(\widehat{u})\| - \|\vec{g}_{\vec{\delta}}\|}$

backing-up



Remarks and conclusions

Remarks: How to apply?

- Our code currently does not calculate the slip on this nodal basis; Rather we compute the numerator and denominator over all slave nodes and evaluate a single scalar; We have experience with this and it seems to work although upon inspection there does seem to be some danger
- We are evaluating the present approach for the individual slip calculation

Conclusions: We have given a straight forward approach to calculating slip increments using readily available information;