On the 'DSF' and the 'Dreaded Surface Effect'
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What is the DSF?
For Isotropic-Ordinary Materials
It \textit{MIGHT} be a Practical Solution to the Following Problem

The following related aspects contribute to the above mismatch.
- Geometric surface effects
- Nonlocal model (dilatation on surface) and model properties
- Discretization error

This talk is about working towards a simple/practical solution.
Acknowledgements

Thanks for the Help and Support

- **Dave Littlewood** – generally helpful with running *Peridigm* and especially for providing the demonstration problem
- **Stewart Silling** – discussion about the ’DSF’ and approach presented here today.
- **Mike Parks** – *Peridigm* support
Mesh Refinement Study

Horizon is tied to mesh element size $h$: $\delta = 3h$

![Graph showing engineering stress vs. engineering strain for different values of $N$](image)

- $N = 2$: $\delta = 0.1860$
- $N = 3$: $\delta = 0.1240$
- $N = 5$: $\delta = 0.0744$
- $N = 7$: $\delta = 0.0531$

*inches*

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Scalar Force State \[ t = \frac{3K\theta}{m} \omega |\xi| + \alpha \omega e^d \]

\[ e = |Y| - |\xi| \quad \theta = \frac{3}{m} (\omega |\xi|) \cdot e \quad e^d = |Y| - |\xi| - \frac{\theta}{3} |\xi| \]

Substituting the above relations

\[ t = \left( \frac{3K}{m} - \frac{\alpha}{3} \right) \theta \omega |\xi| + \alpha \omega e \]

If we use \( \alpha = \frac{15\mu}{m} \) (trouble starts here)

\[ t = \left( \frac{9K - 15\mu}{3m} \right) \theta \omega |\xi| + \alpha \omega e \]

We need a ’discretization’ appropriate and ’geometric’ appropriate \( \alpha \)
infinitesimal shear deformation
St. Venant-Kirchhoff Material

Displacements

\[ u = \gamma y \]
\[ v = \gamma x \]
\[ w = 0 \]

Deformation Gradient \( F \)

\[
[F] = \begin{bmatrix}
1 & \gamma & 0 \\
\gamma & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Green Lagrange-Strain \( E \)

\[
2[E] = [F]^t[F] - I = \begin{bmatrix}
\gamma^2 & 2\gamma & 0 \\
2\gamma & \gamma^2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Strain energy density \( W_L = \frac{1}{2} \lambda [tr(E)]^2 + \mu E : E = \frac{1}{2} \lambda \gamma^4 + 2\mu \gamma^2 \)
infinitesimal shear deformation
Ordinary Isotropic Peridynamic Material

On a sphere

Deformation State $Y$

$Y(\gamma) = \sqrt{z^2 + (y + \gamma x)^2 + (x + \gamma y)^2}$

Linearize deformation state

$Y(\gamma) \approx \ldots + \mathcal{O}(\gamma^2)$

Form extension state $e = Y - |\xi|$

$e = \gamma |\xi| \sin^2(\theta) \sin(2\phi)$

Verify dilatation is zero

$\frac{3}{m} |\xi| \cdot e = 0$

Weighted volume on a sphere

$m = \frac{4\pi \delta^5}{5}$

Energy density $W = \frac{1}{2} \alpha e^d \cdot e^d = 2\alpha \frac{m\gamma^2}{15}$
Elastic energy density for *infinitesimal* shear deformation

Review Stewart’s Approach

Equating nonlocal $W$ with local $W_L$

On a sphere

$$W = \frac{1}{2} \alpha e^d \cdot e^d = 2\alpha \frac{m\gamma^2}{15}$$

$$W_L \approx \mu \gamma^2$$

$$W_L = W \implies \alpha = \frac{15\mu}{m}$$

Observations
- $W_L$ is independent of position, i.e., constant
- $W$ depends upon neighborhood through integral.
Elastic energy density

Consider simple shear: $u = \tilde{\gamma} y; \quad v = 0; \quad w = 0; \quad \gamma = \tilde{\gamma}; \quad W_L = \frac{1}{2} \mu \tilde{\gamma}^2$

$\mu = 6.923 \times 10^{11}; \quad K = 1.5 \times 10^{12}; \quad \tilde{\gamma} = 1.0 \times 10^{-6}; \quad W_L \approx .34615$
Motivated by Stewart’s Approach
Create a Surface Correction Factor: \( DSF \)

Reconsider elastic energy density for shear at a point

\[
W = \frac{1}{2} \tilde{\alpha} \| e^d \|_h^2 = W_L = 2\mu \gamma^2
\]

Divide both sides by \( \| e^d \|_h^2 = \frac{4m\gamma^2}{15} \), where \( m = \frac{4\pi\delta^5}{5} \)

\[
\tilde{\alpha} = \frac{\| e^d \|_h^2 15\mu}{\| e^d \|_h^2 m}
\]

\[
= \frac{4m}{15} \gamma^2 \frac{15\mu}{m}
\]

\[
= DSF \times \alpha
\]
Consider simple shear: \( u = \gamma y; \quad v = 0; \quad w = 0; \quad \gamma = \frac{\gamma}{2}; \quad W_L = \frac{1}{2}\mu \gamma^2 \)

\[ \mu = 6.923 \times 10^{11}; \quad K = 1.5 \times 10^{12}; \quad \dot{\gamma} = 1.0 \times 10^{-6}; \quad W_L \approx .34615 \]

\[ \alpha = \frac{15\mu}{m}; \quad \tilde{\alpha} = DSF \times \alpha \]
Mesh Refinement Study w/DSF
Horizon is tied to mesh element size $h$: $\delta = 3h$
Algorithm for computing the **DSF**

*For all* points $XP$ in the neighborhood of $X$

**Apply pure shear deformation**

```c
double dx = XP[0] - X[0];
double dy = XP[1] - X[1];
double dz = XP[2] - X[2];
double zx(0.0), xy(0.0), yz(0.0);
double xz(0.0), yx(0.0), zy(0.0);
switch (mode) {
  case ZX:
    zx = gamma * dx;
    xz = gamma * dz;
    break;
  case XY:
    xy = gamma * dy;
    yx = gamma * dx;
    break;
  case YZ:
    yz = gamma * dz;
    zy = gamma * dy;
    break;
}
YP[0] = XP[0] + xy + xz;
```

**Compute DSF**

```c
double scf, max_dsf, DSF;
int mode = XY;
Y[0] = X[0]; Y[1] = X[1]; Y[2] = X[2];
apply_pure_shear(X, Y, XP, YP, mode, gamma);
scf = ed_squared(neigh[X], X, Y, mode);
max_dsf = scf;
mode = ZX;
Y[0] = X[0]; Y[1] = X[1]; Y[2] = X[2];
apply_pure_shear(X, Y, XP, YP, mode, gamma);
scf = ed_squared(neigh[X], X, Y, mode);
if (scf > max_dsf) max_dsf = scf;
mode = YZ;
Y[0] = X[0]; Y[1] = X[1]; Y[2] = X[2];
apply_pure_shear(X, Y, XP, YP, mode, gamma);
scf = ed_squared(neigh[X], X, Y, mode);
if (scf > max_dsf) max_dsf = scf;
scf = max_dsf;
DSF = 4.0 * gamma * gamma * m / scf / 15.0;
```

Computation on right uses *max*
Mesh Refinement Study w/DSF
Horizon is tied to mesh element size $h$: $\delta = 3h$
Mesh Refinement Study w/DSF
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Mesh Refinement Study w/DSF
Horizon is tied to mesh element size $h$: $\delta = 3h$
**infinitesimal dilatation**

St. Venant-Kirchhoff Material

Displacements

\[ u = \Delta x \]
\[ v = \Delta y \]
\[ w = \Delta z \]

**Deformation Gradient** \( F \)

\[
[F] = \begin{bmatrix}
1 + \Delta & 0 & 0 \\
0 & 1 + \Delta & 0 \\
0 & 0 & 1 + \Delta \\
\end{bmatrix}
\]

**Green Lagrange-Strain** \( E \)

\[
2[E] = [F]'[F] - I
\]
\[
= \begin{bmatrix}
\Delta^2 + 2\Delta & 0 & 0 \\
0 & \Delta^2 + 2\Delta & 0 \\
0 & 0 & \Delta^2 + 2\Delta \\
\end{bmatrix}
\]

*Infinitesimal* assumption: retain only \( \Delta^2 \) terms

Strain energy density \( W_L = \frac{1}{2} \lambda [tr(E)]^2 + \mu E : E = \frac{1}{2} (9\lambda + 6\mu) \Delta^2 \)
Expectation for isotropic ordinary materials

- Recover *Youngs Modulus* on a relatively *coarse* mesh

Using an energy based approach

- Developed shear modulus correction that shows promise
- Speculate that practical/ideal ’DSF’ exists
- Question: Is the ideal ’DSF’ *anisotropic* at the surface?

Identified issue

- Nonzero dilatation on surface under conditions of pure shear

Surface effect diminishes with mesh refinement

- Convergence is slow but apparent
- On *tensile test*, rate of convergence is inadequate