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Foundations of Generalized Reversible Computing

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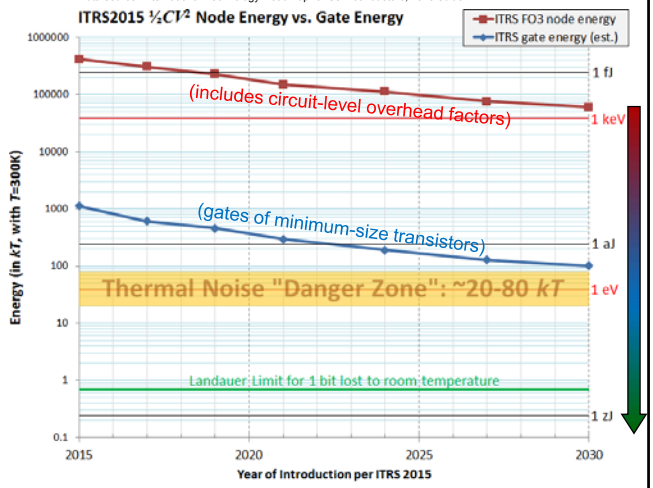
Talk Outline

- Motivation
  - Sustaining computer efficiency gains will require practical reversible HW
  - Modeling all reversible HW requires a more general theoretical foundation
- Fundamental physical foundations
  - Information/entropy, bijective dynamics, conditional entropy
- Formulating Landauer's Principle
  - A rigorous quantitative derivation from fundamental physics
- Redeveloping reversible computing theory from first principles
  - Traditional theory of unconditionally reversible computing
    - *Fundamental theorem of traditional reversible computing*
  - General theory of conditionally reversible computing
    - *Fundamental theorem of generalized reversible computing*
- Applications of Generalized Reversible Computing (GRC) theory
  - Examples of conditioned reversible operations
  - Modeling reversible hardware (*e.g.*, adiabatic circuits)
- Conclusion

Semiconductor Roadmap is Ending!

- Thermal noise on minimum-width segments of FET gate electrodes leads to channel fluctuations below ~1-2 eV
  - Increases leakage, impairs device performance
    - Thus, ITRS has minimum gate energy asymptoting to ~2 eV
- Also, real logic circuits incur many further overhead factors:
  - Transistor width 10-20× min.
  - Parasitic (junction, etc.) transistor capacitances (~2×)
  - Multiple (~2) transistors fed by each input to a given logic gate
  - Fan-out to a few (~3) logic gates
  - Parasitic wire capacitance (~2×)
- Due to all these overheads, the energy of each bit in real logic circuits is many times larger than the min.-width gate energy
  - 375-600× (!) larger in ITRS'15
    - ∴ Practical bit energy for irreversible logic asymptotes to **~1 keV**!
- Practical, real-world logic circuit designs can't just magically cross this ~500× architectural gap!
  - ∴ Thermodynamic limits imply much *larger* practical limits!
    - This is good news for our field…

Data source: International Technology Roadmap for Semiconductors, 2015 edition



Only reversible computing can take us from ~1 keV at the end of the CMOS roadmap, all the way down to $\ll kT$!

Motivation for this work

- To make reversible computing practical will be essential for sustaining the exponential growth of computer energy-efficiency over multi-decade timeframes, looking forward…
  - If we are successful at this, it can be reasonably expected to have a correspondingly enormous impact on future economic growth as well
    - Thus, there is a case for major investment, ***but*** we need a solid foundation!
- Unfortunately, the traditional (Landauer-Fredkin-Toffoli) theory of reversible logic circuits is not really adequate as a foundation for the design of real (physically) reversible hardware,
  - because it is *insufficiently expressive* to describe and explain the reversibility (at both the logical & physical levels) of the simplest real reversible hardware devices that we can build
    - We will see a number of examples and proofs of this later…
- Thus, we need a new, more general (and more expressive) theoretical foundation, which is suitable to serve as the basis for modeling any and all real reversible hardware devices & circuits…
  - We begin by developing a simple, deterministic framework for this…
    - Later, we can extend it to stochastic and/or quantum variants

Fundamental physical foundations

- It's widely known that Landauer's Principle (that information loss implies energy dissipation) is the rationale for studying reversible computing as a basis for energy efficiency gains…
  - But, it's perhaps less widely-understood how Landauer's Principle *itself* can be rigorously proved from more fundamental physics.
- We'll briefly review the following fundamental physical concepts that are necessary to fully understand the foundations of the thermodynamics of computation:
  - ***Entropy*** – What is it, how to understand its definition
  - ***Bijective dynamics*** – A core property of fundamental physics
  - ***Physical* vs. *computational* states** – And entropic implications
  - ***Conditional entropy*** – A visual illustration of its basic properties
- This will then set us up to rigorously derive a detailed quantitative form of Landauer's Principle…

Entropy in a Nutshell

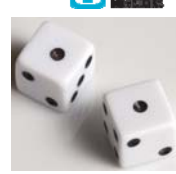
Basic review + coining some useful terminology

- Define the "surprisingness" or *surprise* $s(x)$ of any event x that has a 1 in m chance of occurring as $s = s(x) = s(m) = \log m$.
  - Call the $m \geq 1$ "improbability;" it can be a non-integer.
    - s is logarithmic b/c the improbabilities of independent surprises multiply.
  - *Indefinite* logarithm; dimensioned in *arbitrary* logarithmic units.
    - Some example units: $\log 2 = 1$ bit; $\log \mathrm{e} = 1$ nat $= k_{\mathrm{B}}$; $\log 10 = 1$ bel.
- In terms of event's *probability* $p = p(x) = p(m) = 1/m$,

$$s(p) = \log \frac{1}{p} = -\log p\,.$$

- Define event's "*heaviness*" $h = h(x) = h(p)$ (Hopefulness? Horribleness?) as its surprise, weighted by its probability:

$$h(p) = s/m = p \cdot s = p \log m = -p \log p.$$



Improbability: $m = 6 \times 6 = 36$

Surprise: $s = 2(\log 6)$

Heaviness: $h = \frac{s}{m} = \frac{2}{36} \log 6$

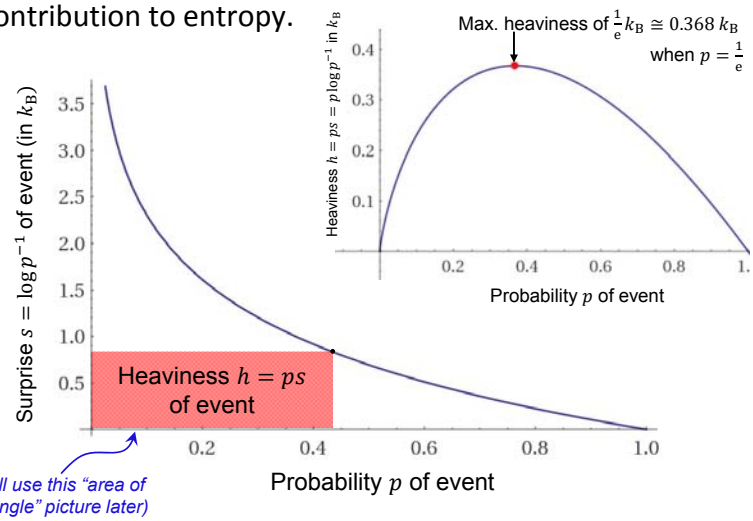
- Then for any probability distribution $p(x)$ over any mutually exclusive and exhaustive set of events $X = \{x_1, \ldots, x_n\}$, we have that the ***expected surprise*** $S(X) = \mathrm{E}_p[s(x)]$ and the ***total heaviness*** $H(X) = \sum_{x \in X} h(x)$ associated with that particular set of possible events are the same, and are given by:

$$S(X) = \sum_{x \in X} p(x) \cdot s(x) = H(X) = -\sum_{x \in X} p(x) \cdot \log p(x)\,.$$

- We call this quantity $H = S$ the *entropy* of the given epistemic situation.
  - By convention, we'll prefer H for "computational" entropy, S for "physical" entropy.

Surprise and Heaviness Functions

- For an individual state's contribution to entropy.

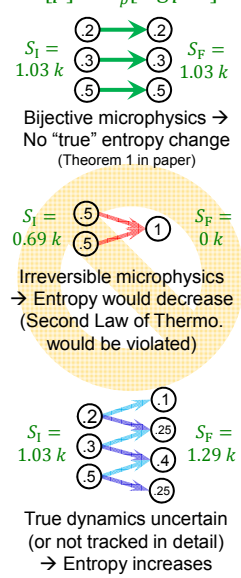


(We'll use this "area of rectangle" picture later)

Thermodynamics and Information

- *Physical* entropy quantifies *uncertainty about the detailed microstate of a physical system*.
  - First postulated by Boltzmann (in his H-theorem)
  - Integral to modern physics (Von Neumann entropy)
  - Depends on modeler's state of knowledge (Jaynes)
- The *reversibility (injectivity)* of microphysics <u>underlies</u> the Second Law of Thermodynamics.
  - States cannot merge as they evolve…
  - Thus, entropy of a closed system cannot decrease!
    - Conserved by unitary quantum time-evolution.
  - Entropy can *increase* if we have any uncertainty about the dynamics, or do not track it in detail
- At the most fundamental level, physical information *cannot be destroyed*.
  - Only *reversibly* transformed, and/or transferred between different subsystems…

$$S[p] = \mathrm{E}_p[\log p^{-1}]$$

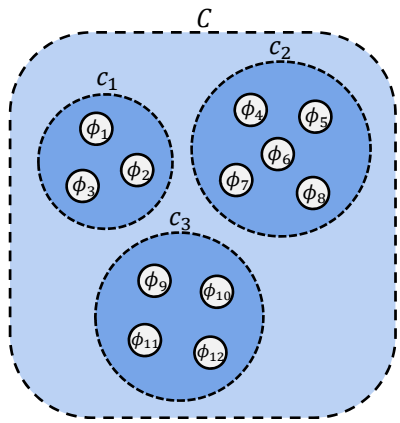


From Physics to Computation

- Thermodynamics and quantum mechanics show that any bounded physical system admits only a finite set $\Phi = \{\phi_1, \dots, \phi_n\}$ of measurably distinguishable detailed physical states (*microstates*).
  - *E.g.*, Φ could be any orthogonal set of basis vectors for the system's Hilbert space.
- We can *group* these microstates, that is, partition them into subsets c_j of micro-states that we consider as *equivalent* to each other for some designated purpose…
  - *e.g.*, for purposes of representing some specific *computational* information
- Any probability distribution $p(\phi_i)$ over the physical state space Φ induces a probability distribution P over the computational state space (subsystem) $C = \{c_j\}$ as well…

$$P(c_j) = \sum_{\phi_i \in c_j} p(\phi_i).$$

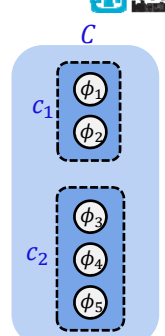
- This implies a *computational entropy* $H(C)$.

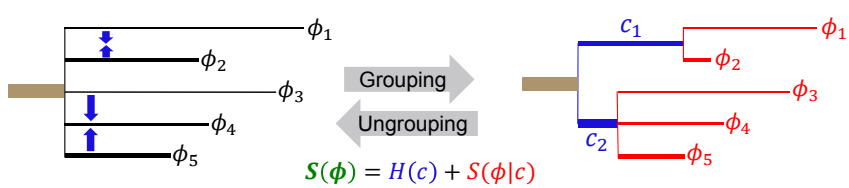


Example of a computational state space C consisting of 3 distinct computational states c_1, c_2, c_3, each defined as a set of equivalent physical states.

Visualizing Entropy of Grouped States

- Can represent a hierarchy of events in a tree structure…
  - Branch ***thickness*** = event probability p.
  - Branch ***length*** = *incremental surprise* Δs associated w. event,
    - relative to whatever base event it's branching off from.
  - Branch ***area*** = event's *incremental heaviness* $\Delta h = p\Delta s$, *i.e.*,
    - its contribution to total entropy, in addition to its base event's.
- *Grouping* events into larger events has these effects:
  - Thicknesses (probs.) of branches combine in parent branch
  - A corresponding part of the total length (surprise) of each branch is reassociated to parent (stem) branch.
  - Note: The total heaviness $\boldsymbol{H}$ of all branches and stems (total entropy $\boldsymbol{S}$) is not changed at all by any grouping/ungrouping!!

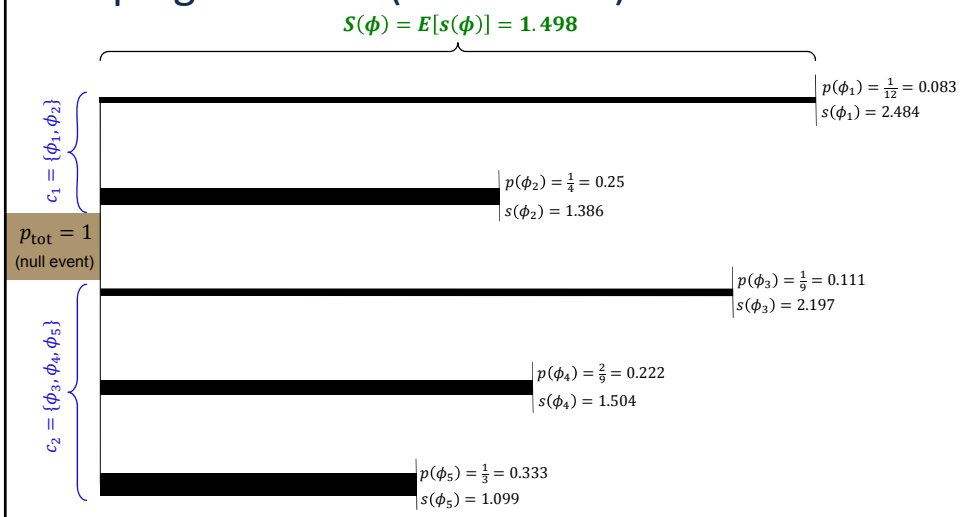




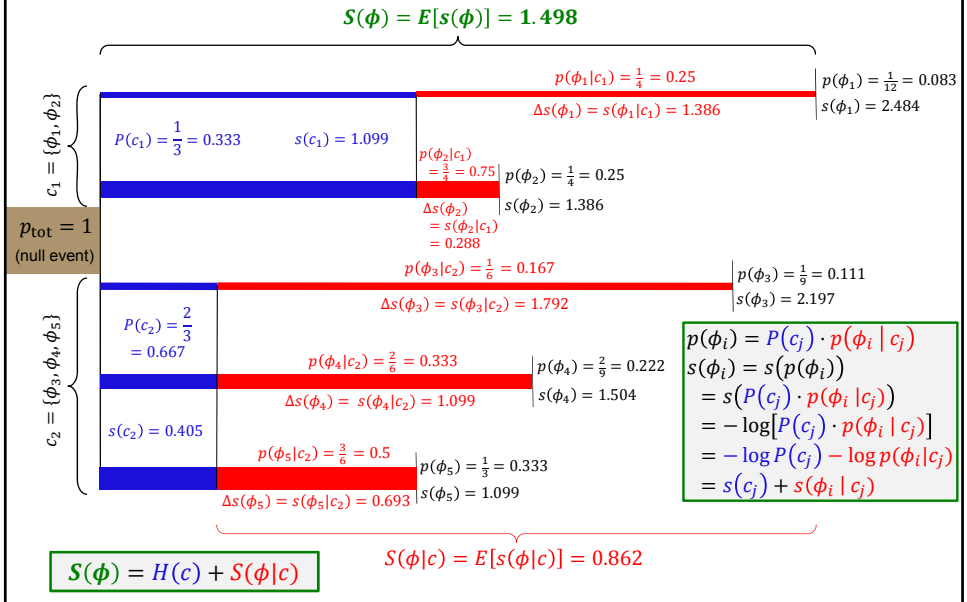
$$\boldsymbol{S}(\boldsymbol{\phi}) = H(c) + S(\phi|c)$$

Total system entropy = computational entropy + non-computational entropy

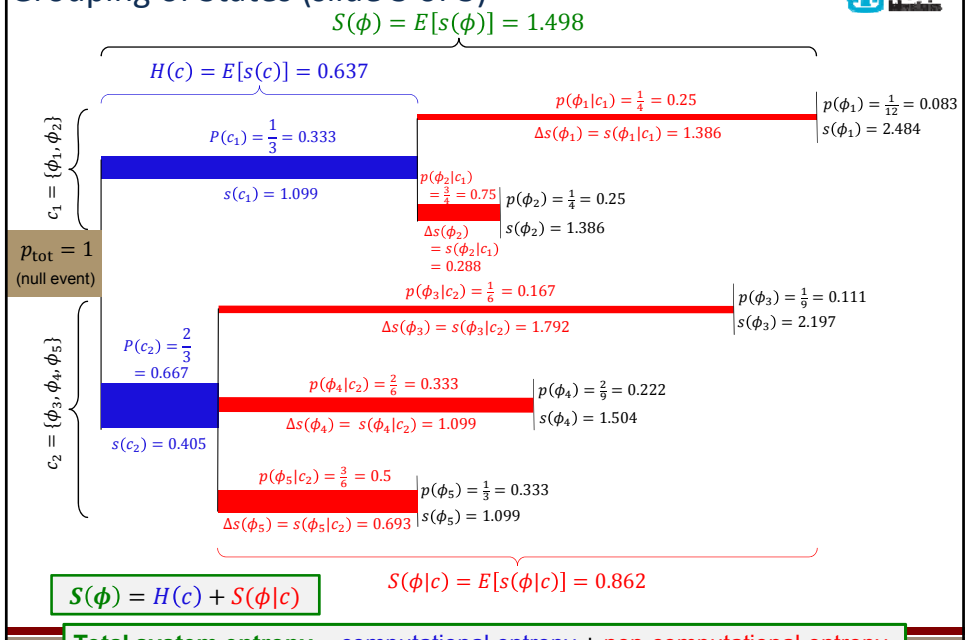
Grouping of States (slide 1 of 3)



Grouping of States (slide 2 of 3)



Grouping of States (slide 3 of 3)



$\boldsymbol{S(\phi)} = H(c) + S(\phi|c)$

Total system entropy = computational entropy + non-computational entropy

Proof of Landauer's Limit

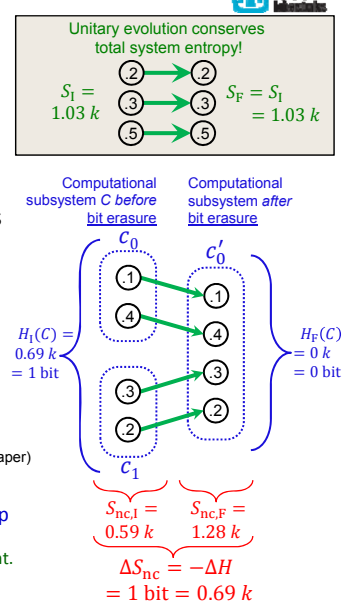
- We've seen that the total system entropy $S(\phi)$ for a given closed system cannot decrease at all…
  - So, what happens if we merge two *computational* states?
- Underlying probability distributions remain the same!
  - Only the identities of the physical states ϕ, and their groupings into computational states, can be changing
- Merging two computational states implies, removing a conceptual partition between groups of physical states
  - Same as the "ungrouping" operation we saw earlier
- The computational contribution $H(C)$ to the total entropy $\boldsymbol{S(\phi)}$ *cannot* simply vanish from existence…
  - Thus, it can only be *ejected* from the **computational** state into the ***non*-computational** state
- We define *non-computational entropy* as:

$$S_{\text{nc}}(\phi) = S(\phi|C) = \boldsymbol{S(\phi)} - H(C).$$

  - So, the *change* in $S_{\text{nc}}(\phi)$ from a merge operation is thus:

$$\Delta S_{\text{nc}}(\phi) = \Delta S(\phi|C) = -\Delta H(C).$$ (Theorem 2 in paper)

- To extent that "non-computational" = "uncontrolled,"
  - the extra non-computational entropy must ultimately end up in some thermal environment at some temperature T
    - We must thus emit at least heat $\Delta Q = T\Delta S$ to that environment. If $\Delta S = 1\text{ b} = k \ln 2$, then $\Delta Q = kT \ln 2$.



∴ **Landauer Limit**: $E_{\text{diss}} \geq kT \ln 2$ per bit lost. ■ (Corrollary 1 in paper)

Redeveloping Reversible Computing Theory

- We can characterize our task, in the development of the most fundamental aspects of Reversible Computing Theory, as follows:
  - We are given, as our starting point, this correct quantitative expression of Landauer's Principle, which follows rigorously from fundamental physics:

    $$\Delta S_{\mathrm{nc}}(\phi) = -\Delta H(C),$$

    - *i.e.*, for any computational operation, the change in the non-computational entropy of the physical state is exactly the negative of the change in the entropy of the computational state (*entropy ejected* from the computation).
      - Note this is referring the *minimum* non-computational entropy change necessitated by thermodynamics—there may be additional entropy increase from other causes.
  - Our task, then, is to infer, from this starting point, what *exactly* are the constraints on the *logical-level structure* of a computation that are necessary and sufficient conditions for this ***entropy ejected*** by a computation to attain or approach 0 (from above), $\Delta S_{\mathrm{nc}}(\phi) \rightarrow^{+} 0$.
    - This will then constrain the design of our physical mechanisms for computation.
- We can say that the following is the underlying essence of what we really mean (or should mean) when we talk about *logical reversibility*:
  - This phrase is, most generally, intended to express the *necessary and sufficient conditions*, at the logical level, under which the change in computational entropy approaches 0 (from below), $\Delta H(C) \rightarrow^{-} 0$.
    - However, Landauer's original definition of the phrase "logical reversibility" *did not actually accomplish this goal in the most general possible sense.*
      - The conditions he gave were sufficient, but, as we'll see *not necessary*.

Logical Reversibility, per Landauer

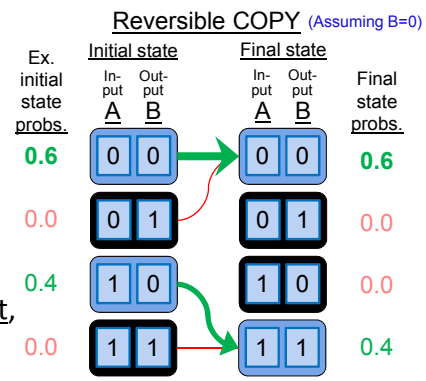
- Here, Landauer defines logical (ir)reversibility for an *N*-bit *device*…
  - Assumes the device is operating on the *entire* space of 2^N *combinatorially possible* initial states (or "inputs")

> … RESTORE TO ONE is an example of a logical truth function which we shall call *irreversible*. We shall call a device *logically irreversible* if the output of a device does not uniquely define the inputs.
>
> … Now assume that the computer is logically reversible. Then the machine cycle maps the 2^N possible initial states of the machine onto the same space of 2^N states, rather than just a subspace thereof.

- But, the probabilities of the initial states are also important!
  - DeBenedictis & Frank previously pointed this out, at the IEEE International Conference on Rebooting Computing 2016 (http://bit.ly/2hYWLdV)
- **Crucial:** If some initial states *have probability 0*, then *not* all of the 2^N combinatorially-possible initial states are *statistically* possible
  - In such operating contexts, a device's operation can transform the full combinatorially describable space of 2^N initial computational states onto a *smaller* set of final states, *while retaining* $\Delta S_{\mathrm{nc}} = 0$ (reversibility)!
    - Landauer's "logical reversibility" tragically obscured this critically important fact!

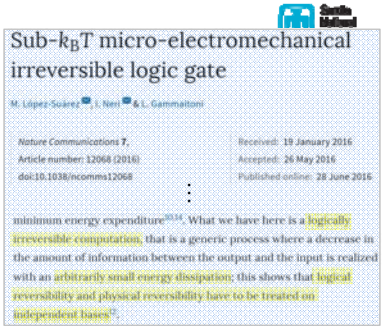
Logically reversible *computations* using "logically irreversible" *devices*

- The operation shown is "logically *ir*reversible" under Landauer's original, literal definition
  - Maps the 2^N=4 initial computational states to only 2 final states!
    - ∴ Merges some states!
- But in this specific operating context, *some initial probabilities are zero*.
  - Under such a distribution, note that the input *is* uniquely determined by the output, given the probabilities!
    - There are $<2^N$ *possible* (*i.e.*, nonzero-probability!) initial states of the device
    - This subset *is* mapped to a (different) set of states with the same size
  - This operation, *done in this context*, *is reversible*, because its $\Delta S_{\mathrm{nc}} = 0$!
    - Doing it does not eject any computational entropy into the environment!



Why this matters…

- Getting the definition of logical reversibility wrong is not without serious consequences!!
  - Example: An article was published in *Nature Communications* last year which claimed to empirically show that even logically irreversible computation doesn't require dissipation (!)


Sub-k_BT micro-electromechanical irreversible logic gate

M. López-Suárez, I. Neri & L. Gammaitoni

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minimum energy expenditure. What we have here is a logically irreversible computation, that is a generic process where a decrease in the amount of information between the output and the input is realized with an arbitrarily small energy dissipation; this shows that logical reversibility and physical reversibility have to be treated on independent bases.

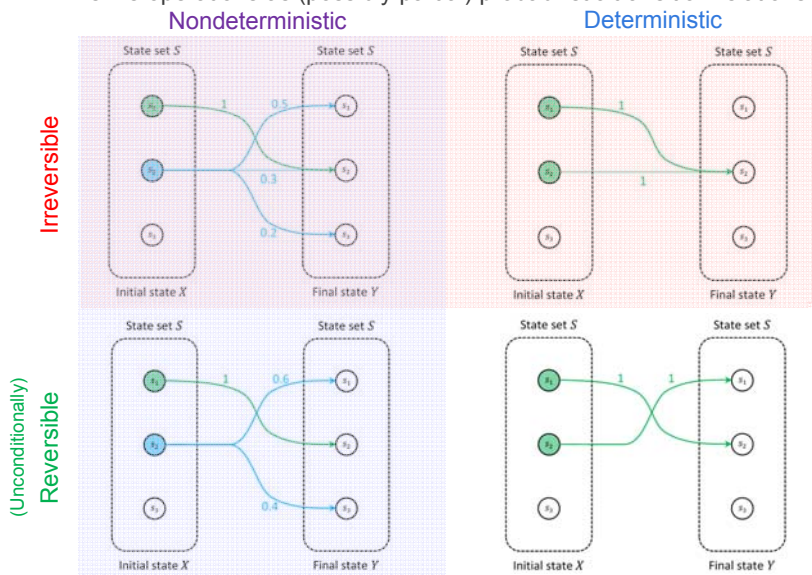
- Of course, the actual computation that they performed was, in fact, *logically reversible*, according to the *correct* definition…
  - It was an example of what I call a *conditioned reversible OR* operation, whose *precondition for reversibility* happened to be satisfied.
- But, we can hardly blame researchers for getting confused about this, when the definition of "logical reversibility" that we ourselves have used since Landauer is in fact *the wrong one*!!
  - Let's get this *right*, and avoid contributing to the widespread confusion…

Devices, Operations, Computations

- To set up the new theory, we'll find it helpful to distinguish several different concepts:
  - ***Device*** – Can perform one (or more) operations.
    - A given device has some associated local state information
      - Includes states of I/O terminals, internal states of device
  - ***Operation*** – a *(computational) operation* is a map O from *initial states* to *final states* (locally)
    - The terms "input" and "output" are too vague – avoid!
    - We can also consider partial maps (undefined=don't-care)
    - Generally speaking, the map O could also be stochastic…
      - Probabilistic transition rule $r_{ij} = \Pr[c_F = c_{Fj} \mid c_{\mathrm{I}} = c_{\mathrm{I}i}]$.
        - » However, that case is not our main focus at present
  - ***Computation*** – a computational operation *performed within a specific operating context*
    - Context specifies/constrains the initial state probabilities
      - These are <u>essential</u> for a meaningful thermodynamic analysis!

Types of Computational Operations

Define operations as (possibly partial) probabilistic transition relations



Operating Contexts & Entropy Ejection

- ***Operating contexts*** - A *(statistical) operating context* for a computational operation O is simply a probability distribution P_{I} over the distinguishable initial computational states of O.
  - A statistical situation in which the operation may be performed.
  - It carries an associated (computational) entropy $H_{\mathrm{I}} = H(P_{\mathrm{I}})$.
  - After performing the operation O in an operating context P_I, we will generally obtain some new ("final") probability distribution P_F,
    - with an associated final computational entropy $H_{\mathrm{F}} = H(P_{\mathrm{F}})$.

$$P_{\mathrm{F}}(c_{\mathrm{F}j}) = \sum_{i=1}^{|C_{\mathrm{I}}|} P_{\mathrm{I}}(c_{\mathrm{I}i}) \cdot r_{ij}$$

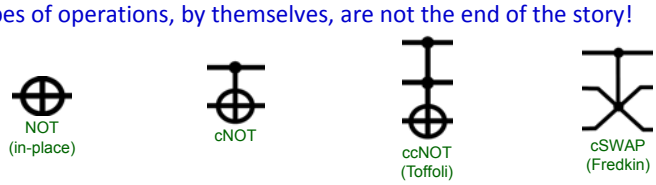
- ***Entropy-ejecting operations*** – A computational operation O is called *(potentially) entropy-ejecting* if and only if there is *some* operating context P_{I} such that, when the operation O is performed in that context, the entropy ejected from computational to non-computational form is positive,

$$\Delta S_{\mathrm{nc}} = -\Delta H = H_{\mathrm{I}} - H_{\mathrm{F}} > 0.$$

  - By convention, we will only call an operation O *non-entropy-ejecting* if (and only if) it is not even *potentially* entropy-ejecting.

Fundamental Theorem of Traditional Reversible Computing

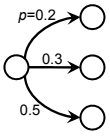
- **Theorem:** *(Theorem 3 in paper)* For any deterministic computational operation O, we have that O is non-entropy-ejecting (according to the prior definition) if and only if O is (unconditionally) reversible (*i.e.*, injective on its entire domain).
  - Proof can be carried out by considering what is the effect on computational entropy when two computational states w. nonzero probs. are merged.
    - However… What happens if a state with *zero* probability is merged with another?
      - This thought sets us up for GRC.
- There are of course many examples of deterministic, unconditionally-reversible operations; these are already very well-known in our field…
  - However, when it comes to *Generalized* Reversible Computing Theory, these types of operations, by themselves, are not the end of the story!



Conditionally Reversible Operations

- Focus here still on *deterministic* operations…
  - Could extend later to nondeterministic (randomizing) ops
- **Definition:** A (deterministic) computational operation O is called *conditionally (logically) reversible* if and only if there is *any* non-empty subset $A \subseteq \mathrm{dom}(O)$ of initial states (called an *assumed set*) that O maps one-to-one onto an equal-sized set of final states.
  - We can say, of any such A, that it is a *sufficient precondition for the (logical) reversibility of* O, or that it is *admissible*, for short.
    - We also refer to O_A (the concept of performing O in a context where A is satisfied) as a *conditioned reversible operation*.
- **Theorem:** All operations that are deterministically defined over some non-empty domain are conditionally reversible. (Slightly stronger version of Theorem 4 in paper.)
  - **Proof:** Consider any singleton set A that consists of any one of the initial states for which O is deterministically defined. ■
- **Theorem:** If O is deterministic, and has n reachable final states, then there's at least one admissible set $A \subseteq \mathrm{dom}(O)$ such that $|A| = n$.

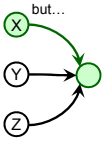
Nondeterministic



Final state is random

Deterministic

This operation is not reversible, but…



It's conditionally reversible, under the condition that the initial state is (say) X

Defining *computations*, and what entropy-ejection and logical reversibility mean for them

- ***Computations*** - A (deterministic) *computation* $\mathcal{C}$ performable by a device D is defined by a *pair* (O, P_{I}) of a deterministic operation O that D can do, *and* an operating context P_{I} for that operation.
  - This pair represents, performing the operation O *within the context* P_{I}.
  - O must be defined over at least all the nonzero-probability initial states
- ***Entropy-ejecting computations*** – A computation $\mathcal{C} = (O, P_{\mathrm{I}})$ is *(specifically) entropy-ejecting* if and only if, when O is performed within the *specific* operating context P_{I}, the entropy ejected from computational to non-computational form is positive, $\Delta S_{\mathrm{nc}} > 0$.
  - A computation (as opposed to an operation) shall be called *non-entropy-ejecting* as long as it is not *specifically* entropy-ejecting.
- ***Logically reversible computations*** – **(NEW, CORRECT DEFINITION)** A deterministic computation $\mathcal{C} = (O, P_{\mathrm{I}})$ shall be called *(specifically) logically reversible* if and only if the set A of all initial states that are assigned nonzero probability within the operating context P_{I} (a.k.a., the *active set*) is a sufficient precondition for the reversibility of O.

Fundamental Theorem of Generalized Reversible Computing

- **Theorem:** *(Theorem 5 in paper)* For any specific deterministic computation $\mathcal{C} = (O, P_{\mathrm{I}})$, we have that $\mathcal{C}$ is (specifically) non-entropy-ejecting if and only if $\mathcal{C}$ is (specifically!) logically reversible.
  - Note that here, we are using our ***new***, more general, contextualized definition of logical reversibility from the previous slide, ***not*** the traditional, more restrictive, context-independent definition!
    - The proof of the theorem is still similarly trivial, however.
- **Theorem:** *(Theorem 6 in paper)* Let O be any operation that is conditionally reversible under some assumed set A, and let the total probability assigned to A by P_{I} approach 1; then the entropy ΔS_{nc} ejected by the computation $\mathcal{C} = (O, P_{\mathrm{I}})$ approaches 0.
  - Thus, merely *approaching* 0 probability of violating some particular assumed one of the sufficient preconditions for reversibility is a sufficient logical-level condition for asymptotically approaching zero entropy ejection (*i.e.*, enabling an approach to thermodynamic reversibility).

Proof Sketch of Asymptotic Theorem

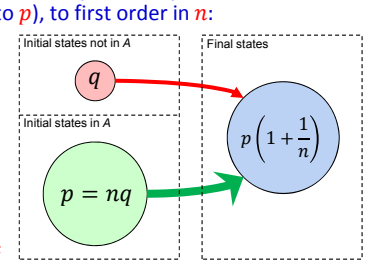
- **Theorem:** Let O be any operation that is conditionally reversible under some assumed set $A \subseteq \mathrm{dom}(O)$, and consider any progression $P_{\mathrm{I}1}, P_{\mathrm{I}2}, \ldots$ of operating contexts in which the total probability assigned to A by $P_{\mathrm{I}i}$ approaches 1 as $i \to \infty$; then the entropy ΔS_{nc} ejected by the computation $\mathcal{C} = (O, P_{\mathrm{I}i})$ approaches 0 in that progression, as $i \to \infty$.
  - By assumption, the sum of the probabilities of states not in A (that is, in the complementary set $B = \bar{A} = \mathrm{dom}(O) - A$) falls to 0:

$$\sum_{c \in B} P_{\mathrm{I}i}(c) \to 0 \text{ as } i \to \infty.$$

  - **Lemma:** For any state $c \in B$ with any probability $q = P_{\mathrm{I}i}(c) > 0$ that O merges with some state in A that has a larger probability $p = nq$ (where $n > 1$), the contribution Δs_{nc} of this state merger to the total entropy ΔS_{nc} ejected from the computation approaches the following expression as the probability ratio n increases (*i.e.*, as the probability q falls, relative to p), to first order in n:

$$\Delta s_{\mathrm{nc}} \to \frac{p}{n}(1 + \ln n) k_{\mathrm{B}}$$

    And, this value itself approaches 0, almost in proportion to $q = p/n$ as it falls.
  - Similarly, for states $c \in B$ merging with other states in B, their contributions Δs_{nc} to entropy ejected are upper-bounded by their heaviness $h = q \log q^{-1}$, which approaches 0 as $q \to 0$.
  - Since all the Δs_{nc} fall to 0, so does their sum ΔS_{nc}



Now, we can validly say this:

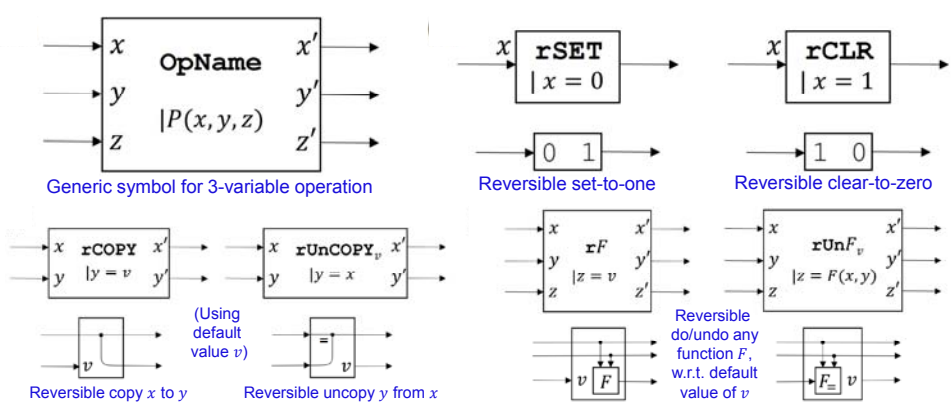
- **Assertion:** For any deterministic computation $\mathcal{C} = (O, P_{\mathrm{I}})$, that computation can be carried out in an asymptotically *thermodynamically* reversible way (within the context of some appropriately-designed family of implementable physical mechanisms) if <u>and only if</u> $\mathcal{C}$ is *specifically* logically reversible
  - …where, note that this assertion invokes our new, corrected definition of logical reversibility.
    - To prove the "if" part of this assertion (and make it a theorem) requires more technology development, but
    - The "only if" part already follows from Landauer's Principle.
- **NOTE:** The classic definition of logical reversibility is the ***<u>wrong one</u>***, because it does *not* actually satisfy (the "only if" part of) the above assertion!!

Notations and Examples

- For conditioned reversible operations; general form:

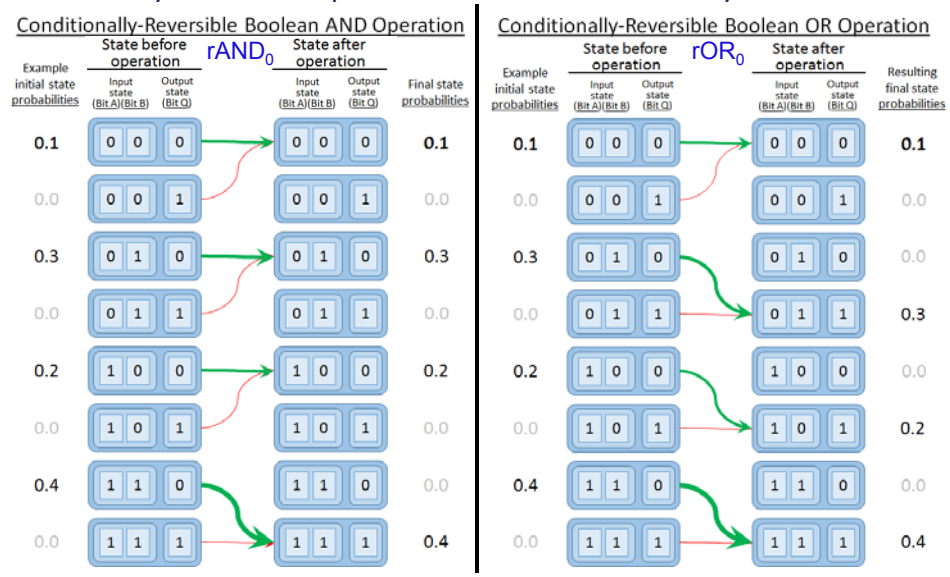
$$\texttt{OpName}(x, y, z \mid P(x, y, z))$$

  - Operation named **OpName**, operating on (typically binary) state variables x, y, z, with a precondition for reversibility of $P(x, y, z)$,
    - *I.e.*, the assumed set of possible initial states is $A = \{(x, y, z) \mid P(x, y, z)\}$.



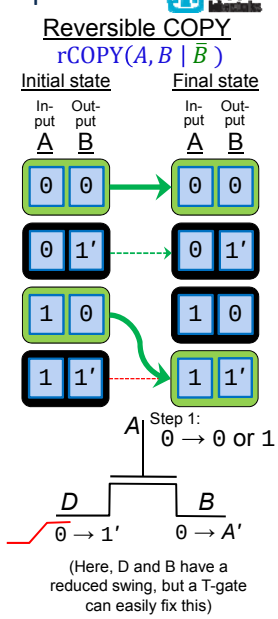
Conditionally-Reversible Operations are Useful!

Universality does *not* require unconditional reversibility



Conditionally-reversible operations can have very simple implementations!

- E.g.: Even *a single MOSFET* (operated adiabatically) can do a certain (conditioned) reversible COPY operation…
  - Operation sequence is as follows:
    0. Driving node D is initially statically held at 0, input *A* also 0.
    1. Input A is externally supplied (D&B connected iff *A* is high)
    2. Externally transition driver D from 0 to (weak) logic high 1′
    3. Voltage level on node B follows D iff A is strong logic high (**1**)
       - B is then afterwards logically equal to A (with a weak swing)
  - Note: Given a (strong) assumed precondition of $\bar{B}$,
    - *i.e.*, if all initial states with $B = 1$ have prob. 0,
    - this indeed performs a reversible COPY operation, $\mathrm{rCOPY}(A, B \mid \bar{B})$.
  - Note: The output in this case is not full-swing,
    - In this diagram, primes (′) denote reduced-voltage logic high signals
      - If we need full-swing outputs, we can use a transmission gate (parallel nFET/pFET pair) with complementary controls
  - A notation precisely describing this operation's semantics is:
    - $[\overline{AB}]$ if $B = 0$ then $B := A$ *(else, leave state unchanged)*
      - The expression $\overline{AB}$ in brackets gives the ***precondition for reversibility*** for the entire operation (the operation is both logically reversible and asymptotically thermodynamically reversible unless $A = B = 1$).
      - The remainder of the statement describes exactly how the state will be transformed in all cases (even if the precondition is not met).
- Note: Traditional reversible computing theory based on unconditionally reversible operations is insufficient to model the logical/physical reversibility of this operation!

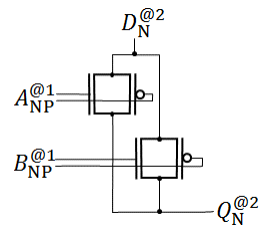


(Here, D and B have a reduced swing, but a T-gate can easily fix this)

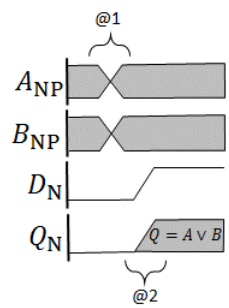
Two-Level Adiabatic Logic (2LAL)

Invented at the University of Florida, circa 2000

- Uses CMOS transmission gates for full-swing switching
  - Logic signals implicitly dual-rail (complementary PN pairs)
  - Logic NOT can be done by simply swapping (relabeling) rails
- Series/parallel T-gate combinations can do Boolean logic…
  - The parallel gate pair at right can be used to (conditionally) reversibly compute the output Q as $A \vee B$ (logic OR)
  - By DeMorgan's law, by complementing both inputs and output of this structure, we can also get: $Q = \overline{\overline{A} \vee \overline{B}} = A \wedge B$ (logic AND).
  - By cascading such gates, we can compute any binary function.
- Operation sequence:
  1. Precondition: Output signal Q is initially at logic 0
  2. Driving signal D is also initially logic 0
  3. At time 1 (@1), inputs A, B transition to new levels
     - Connecting D to Q if and only if A or B is logic 1
  4. At time 2 (@2), driver D transitions from 0 to 1
     - Q follows it to 1 if and only if A or B is logic 1
     - Now Q is the logical OR of inputs A,B
- Additional reversible steps that we can do afterwards:
  - Restore A, B to 0 (latches output Q in place at its current level)
    - 2LAL was the first adiabatic logic family capable of doing both logic and latching in the same structure!
  - Or, simply perform the above operation sequence in reverse



(and similarly for Q_P)



Unconditionally-Reversible Operations are only a special case…

More critiques of Landauer '61…

- It was Landauer who first introduced what's now called a *Toffoli gate operation*, or controlled-controlled-NOT (ccNOT), an unconditionally logically reversible operation:

  $$r := r \oplus pq.$$

  - Landauer describes (correctly) that AND can be embedded into this operation. (Given initial $r = 0$)
- However, his statement here that the AND operation "is not, in itself, reversible" is somewhat misleading!
  - That would only be true if:
    - The input bits were *consumed*…
      - But, in modern technologies such as CMOS, gates never actually consume their inputs!
    - Or, if the output bit was *destructively overwritten* with the result…
      - But, doing that is not necessary either!

> Consider, for example, a particular three-input, three-output device, i.e., a small special purpose computer with three bit positions. Let p, q, and r be the variables before the machine cycle. The particular truth function under consideration is the one which replaces r by $p \cdot q$ if $r = 0$, and replaces r by $\overline{p \cdot q}$ if $r = 1$. The variables p and q are left unchanged during the machine cycle. We can consider r as giving us a choice of program, and p, q as the variables on which the selected program operates. This is a logically reversible device, its output always defines its input uniquely. Nevertheless it is capable of performing an operation such as AND which is not, in itself, reversible.

- The approach Landauer takes here, of XOR'ing the result into the output bit, is indeed one that is logically reversible in *all* operating contexts.
  - But, it is rather complex to implement…
    - The simpler, conditionally-reversible setting of the output (w.r.t. an assumed default value) also works fine, in suitably restricted operating contexts!
- In my opinion, as a community, we should avoid making statements such as "AND and OR are irreversible," since this can be misleading…
  - Unless we include all of the appropriate caveats.

All truly, fully adiabatic circuits are conditionally reversible!

- "Dry switching" rules for designing truly adiabatic circuits:
  - Never close a switch when there's a voltage ≠0 between its terminals
    - *E.g.*, don't turn on a transistor when $V_{\mathrm{DS}} \neq 0$.
  - Never open a switch when there's a current passing through it.
    - *E.g.*, don't turn off a transistor when $I_{\mathrm{DS}} \neq 0$.
      - Only exception to this rule: If there's an alternate path for the current.
  - Never pass current through diodes (which have a voltage drop)
- Violating any of these rules leads to significant dissipation!
- **Theorem:** The operation of a switching circuit carries out a (conditionally) logically reversible computation, in any operation context where the above rules are always satisfied.
  - *It's impossible to erase information in any truly, fully adiabatic logic operation.* → Logically-reversible computing is key to adiabatic design
    - But, the *right* definition of "logically reversible" is our generalized one!
      - The same structures are *not* reversible/adiabatic if the rules are violated

Nondeterministic GRC Theory

- Let's briefly consider now the case of nondeterministic (here meaning, stochastic) computational operations…
  - These tend to increase the entropy in the computational state, that is, we have a positive change in computational entropy, $\Delta H > 0$,
    - Thus, they tend to *decrease* the non-computational entropy, so that we can have a *negative* change in non-computational entropy, $\Delta S_{\mathrm{nc}} < 0$…
      - Operations can be entropy-absorbing, as opposed to entropy-ejecting!
        - » *C.f. paramagnetic cooling* as a real physical example of this effect
- Even if a stochastic computation is logically "irreversible" (in the sense of, not injective over its active set), it could *still* be non-entropy-ejecting, or even entropy-absorbing…
  - As long as the increase in computational entropy due to any nondeterminism at least "breaks even," with respect to counteracting any decrease in computational entropy due to its irreversibility
    - Such cases still need to be analyzed and characterized in more detail

Some ideas for future work…

1. Give more examples of conditioned reversible operations;
   - Find other simple, natural primitives that are easy to implement;
2. Illustrate detailed physical implementations of devices for performing such operations;
3. Further develop the theory for the nondeterministic case;
   - Explore potential applications, *e.g.*, in randomized algorithms
4. Develop additional descriptive frameworks at higher levels, building upon the GRC foundation, *e.g.*:
   - Formal/algebraic representation theory for GRC constructs
     - Notations distinguishing preconditions for correctness *vs.* reversibility
   - GRC-based hardware description languages
     - GRC-based circuit architectures for useful functions
   - GRC-based programming languages
     - GRC-based software algorithms for useful functions

Conclusion

- By examining precisely how and why the validity of Landauer's Principle follows rigorously from fundamental physics,
  - we can clearly infer that there exists a much broader range of computations that can avoid ejecting entropy from the computational to the non-computational state than is traditionally recognized within reversible computing theory.
    - This leads to a corresponding *new definition* of logical reversibility that applies to appropriately-*conditioned* computational operations, as well as to specific computations operating within any statistical contexts that meet those conditions.
- It is the resulting theory of ***Generalized Reversible Computing*** (GRC), and *not* the traditional theory of reversible computing alone, that is the most appropriate foundation for the design of real reversible hardware, because:
  - It well models the (generally only conditionally-reversible!) nature of the reversible hardware devices that we actually know how to build;
    - Such as, for example, adiabatic circuits constructed out of MOSFETs;
  - It broadens the design space for constructing reversible architectures and algorithms,
    - And allows us to compose them out of simpler primitives than we have been using so far;
- Understanding GRC theory also helps resolve some longstanding, widespread confusions about the (correct, but widely misunderstood) connection between logical and physical reversibility, and between information and entropy.
  - We hope that this improved understanding will facilitate the emergence of reversible computing as the dominant foundation for 21st-century computing…
    - Since this will be absolutely required in order for the efficiency and economic impact of computing to continue increasing by many more orders of magnitude

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