Code verification for the eXtended Finite Element Method (XFEM): the compound cohesionless impact problem

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Albuquerque, New Mexico, USA

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Arcachon, France

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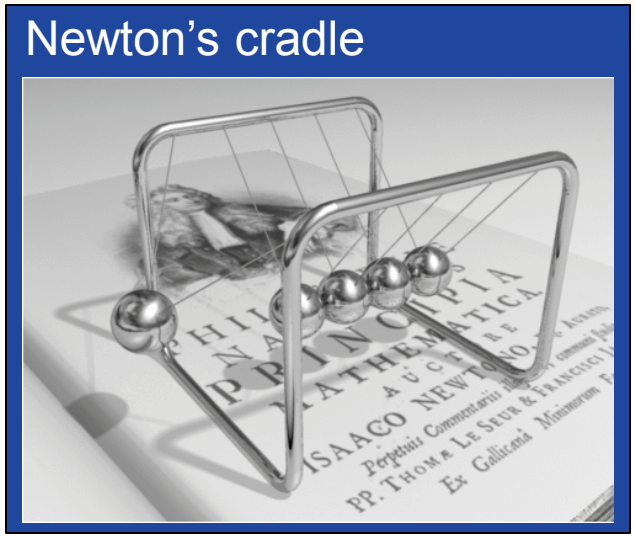
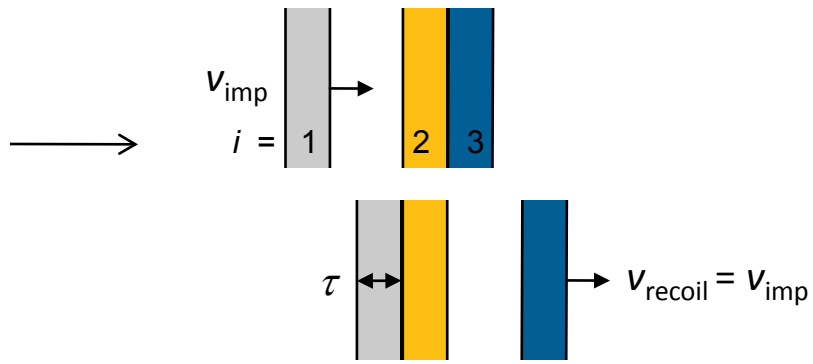
Approved for unlimited release: SAND2011-6135C

A familiar and seemingly simple problem in solid dynamics: “Newton’s cradle”



Animation by Dominique Toussaint (www.edumedia-share.com, GNU)

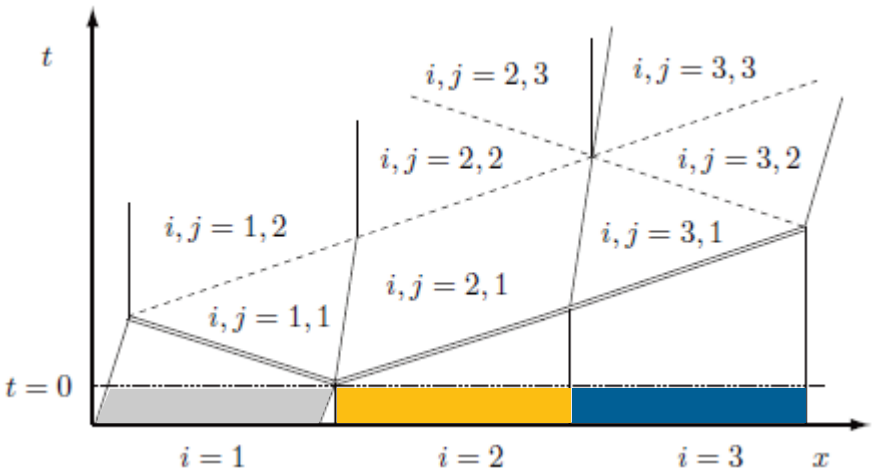
1D cohesionless, compound impact:



In this study we consider in one dimension the impact of a solid projectile on a compound, cohesionless solid target.

An attractive problem for verification:

- Impact and release play important roles.
- Analytic solution is easily obtained…



Analytic solution: $v_{3,3} = v_{1,0} = v_{imp}$

…if dissipation is absent.

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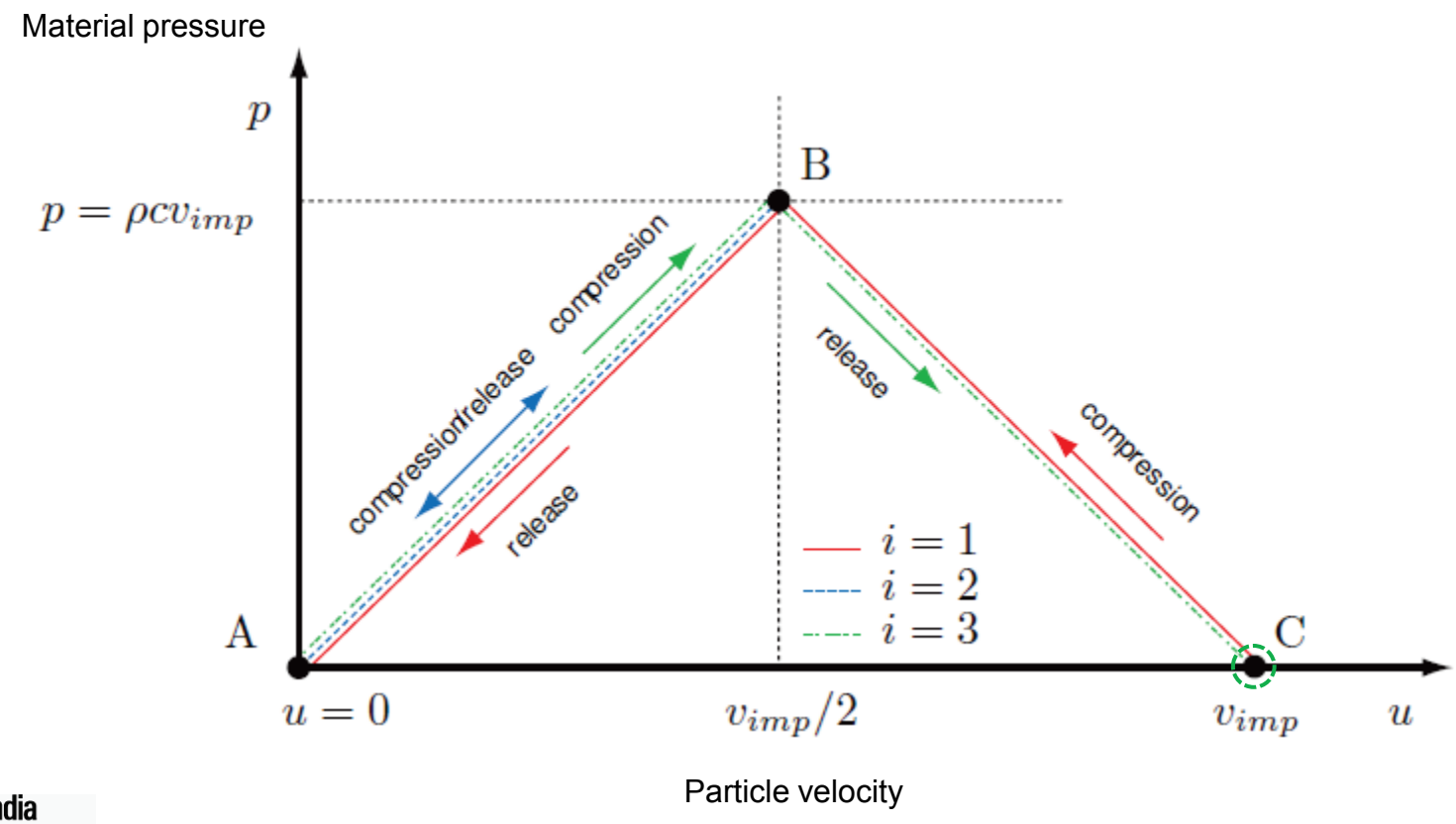
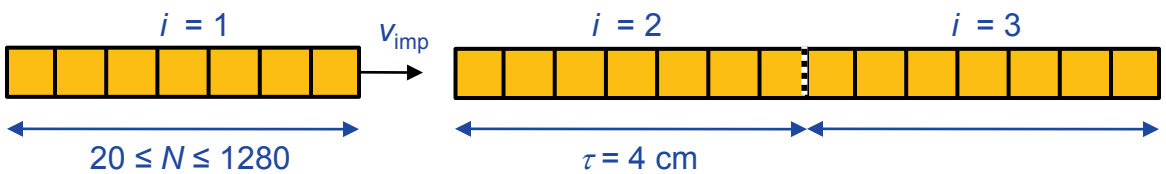
The exact solution can be obtained using a graphical analysis

The analytic solution for the case of symmetric, dissipationless impact can be obtained by a simple graphical analysis. (Full mathematical analysis is discussed in the extra slides.)



A multimaterial shock/release verification problem for ALEGRA was created

The cohesionless impact problem has been set up for **ALEGRA** using copper with a modified Mie-Grüneisen EOS and a linear-elastic strength model.



Material setup

ρ_0 = 8930 kg/m^3
c_0 = 3940 m/s
C_V = 393.1 J/kg/K
$E = \rho_0 c_0^2$

Dissipationless contraints

$\Gamma_0 = 0$
$s = 10^{-12}$
$Q = 0$
$\nu^{HG} = 0$

ALEGRA: a multiphysics, multimaterial, ALE finite-element code maintained at Sandia National Laboratories since the 1990's.

See also:

- T. Voth, Wednesday morning – XFEM
- W. Rider, Tuesday afternoon – Limiting for artificial viscosity
- A. Robinson, Friday afternoon – Uncertain EOS models

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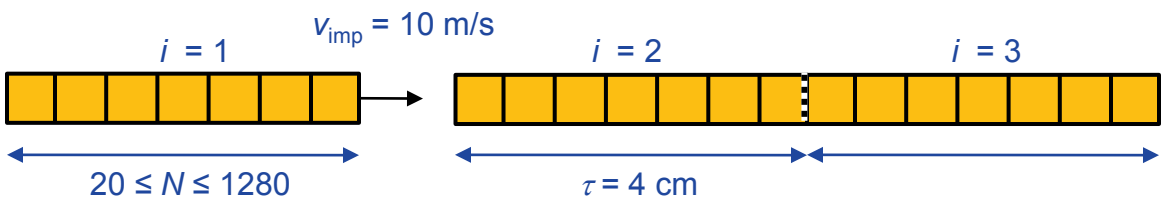
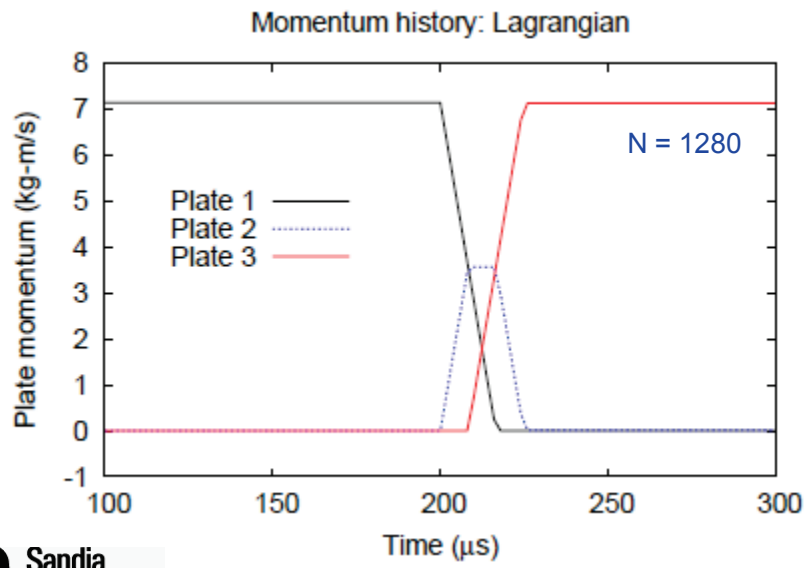
Lagrangian method with contact captures the correct behavior

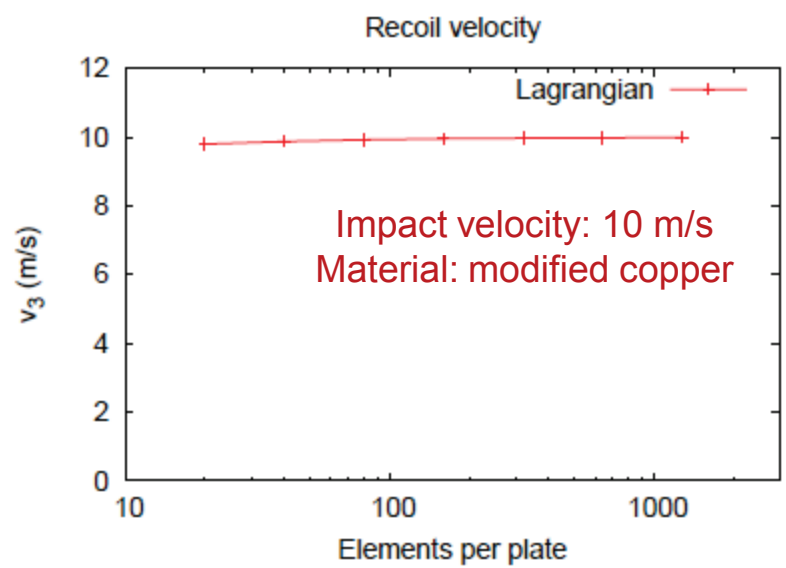
The cohesionless impact problem has been set up for ALEGRA using copper with a modified Mie-Grüneisen EOS and a linear-elastic strength model.



ρ_0 = 8930 kg/m^3
c_0 = 3940 m/s
C_V = 393.1 J/kg/K
Γ_0 = 0
s = 10^{-12}
Q = ν^{HG} = 0

For v_{imp} = 10 m/s, ALEGRA with Lagrangian mesh shows the expected behavior:





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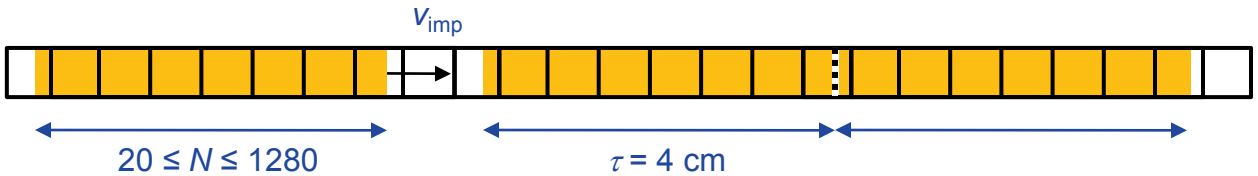
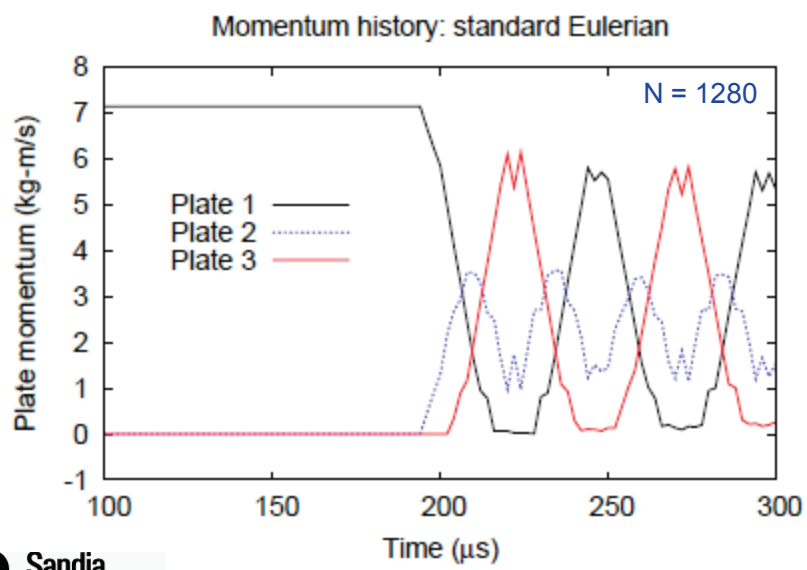
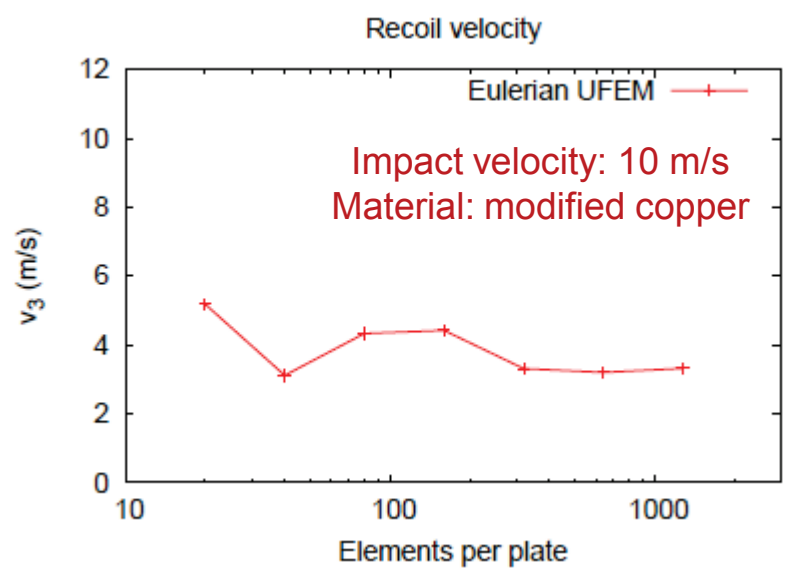
Standard Eulerian FEM fails to capture plate separation

However, in the Eulerian frame, conventional FEM has no native capacity for
(1) Void closure during impact, or
(2) Void opening during recoil



For v_{imp} = 10 m/s, it therefore fails to recover the analytic recoil velocity:





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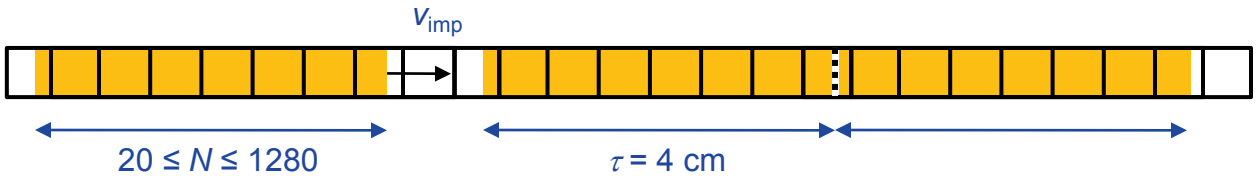
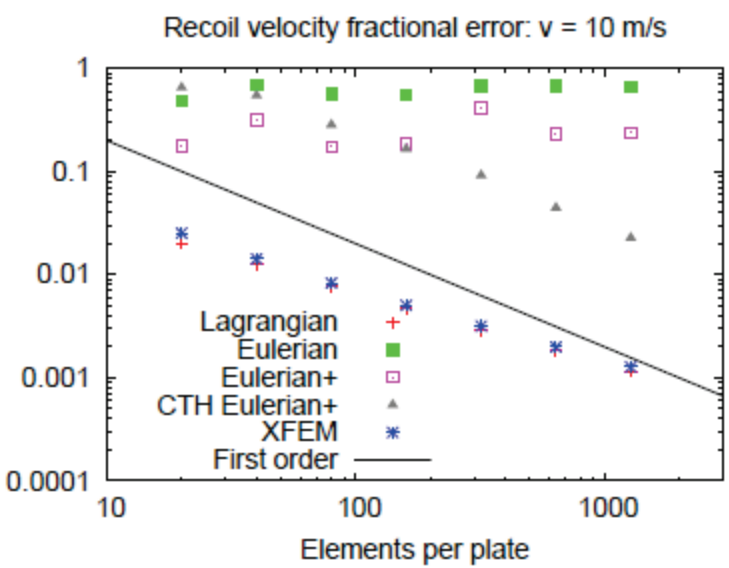


For v_{imp} = 10 m/s, it therefore fails to recover the analytic recoil velocity:

Fractional error:
$\varepsilon = (v_{recoil} - v_{imp}) / v_{imp}$



- Lagrangian: standard unenriched ALEGRA, no remap.
- Eulerian: standard unenriched ALEGRA with remap.
- Eulerian+: activate internal *ad hoc* interface physics modifications.
- CTH Eulerian+: run identical simulation in CTH with ad hoc interface physics modifications (Marlin Kipp).

Eulerian+: Eulerian with "intermaterial fracture" (imposed interfacial material damage)

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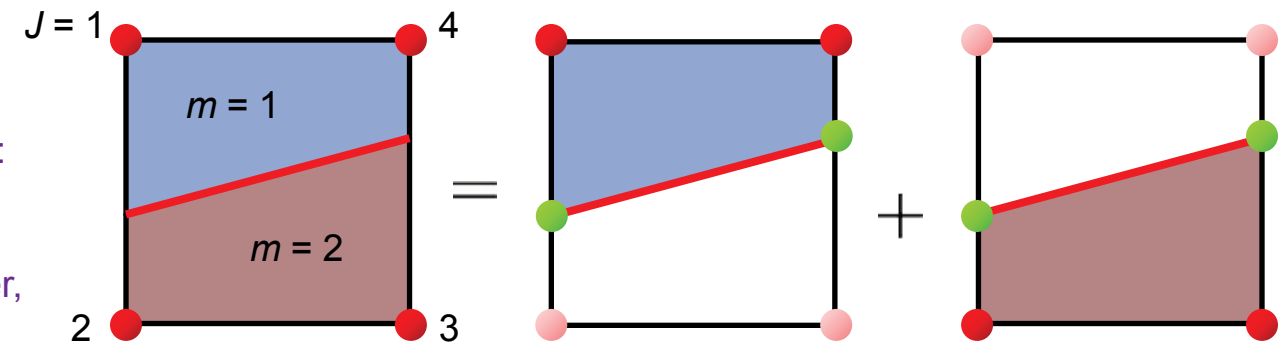
The eXtended Finite Element Method (XFEM): implemented in ALEGRA for multimaterial ALE

XFEM can be used for capturing intra-element interface physics in Eulerian solid mechanics simulations, by element decomposition with the Heaviside function.

ALEGRA developers: T. Voth, J. Mosso, D. Hensinger, C. Luchini



"enriched" elements

J = node index
m = material index

$$u^h(\mathbf{x}) = \cancel{\sum_I u_I^0(\mathbf{x}) N_I(\mathbf{x})} + \sum_m \sum_J u_J^m N_J(\mathbf{x}) H_m(\mathbf{x})$$

$$u^h(\mathbf{x}) = u_A^h(\mathbf{x}) + u_B^h(\mathbf{x})$$

$$u^h(\mathbf{x}) = \sum_J u_J^m N_J(\mathbf{x}) H_m(\mathbf{x})$$

See Voth, *et al.*, "An implementation of the X-FEM for Eulerian solid-mechanics." (Wednesday morning)

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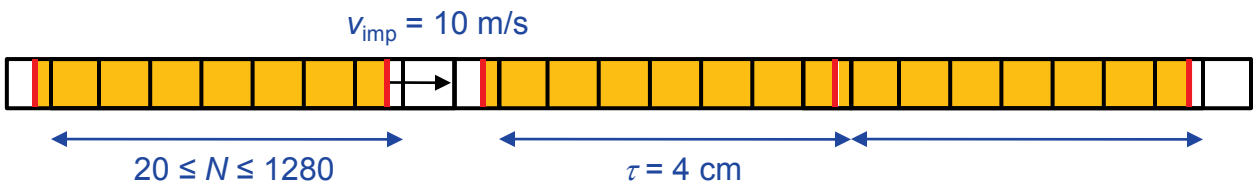
With XFEM, gap closure and opening is captured naturally in the Eulerian frame

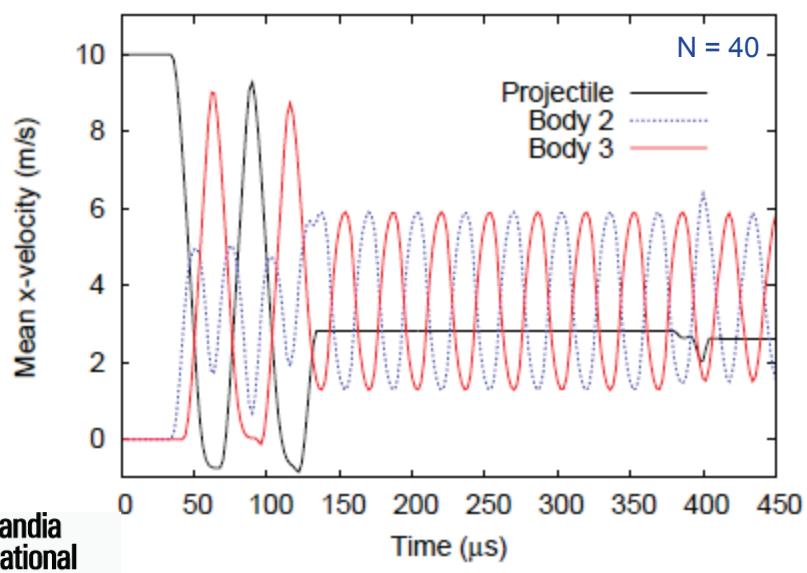
With XFEM, the material interfaces are no longer subgrid.

- Momentum transfer does not begin before full gap closure during impact.
- Gap opening allows recoil to happen naturally.

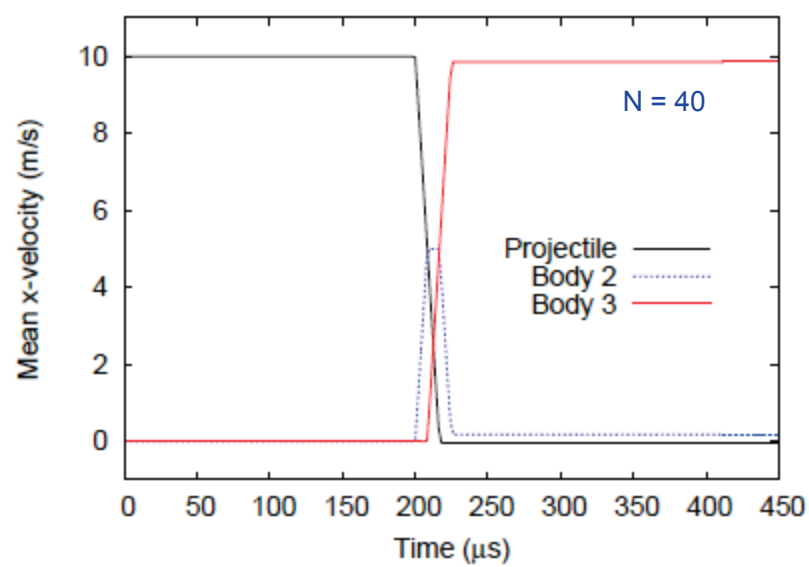


For v_{imp} = 10 m/s, the expected plate recoil is seen in Eulerian frame only with XFEM:

Standard Eulerian FEM



Eulerian XFEM



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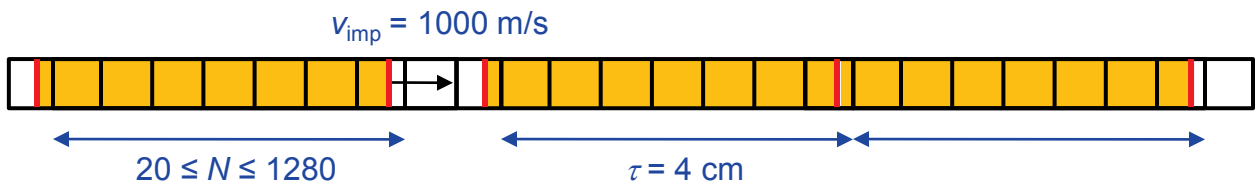
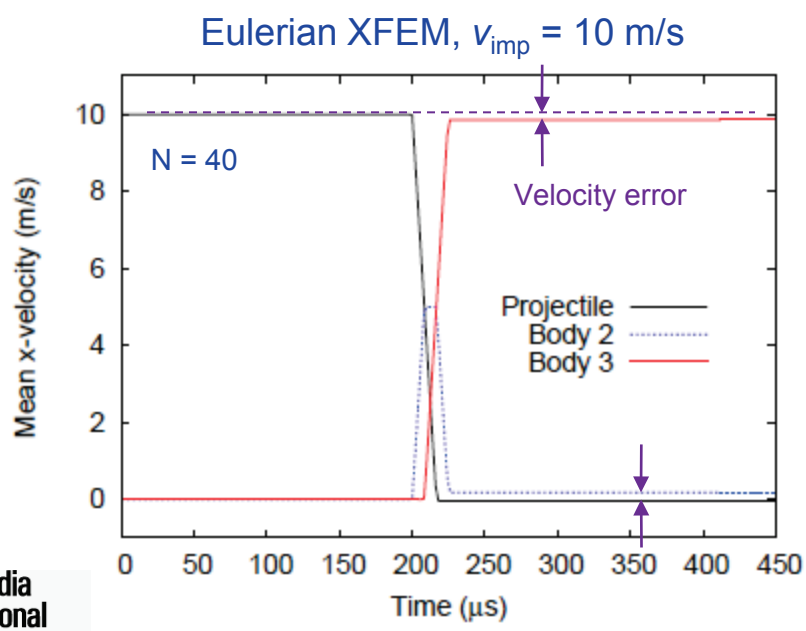
Eulerian XFEM captures plate separation at high and low impact velocity

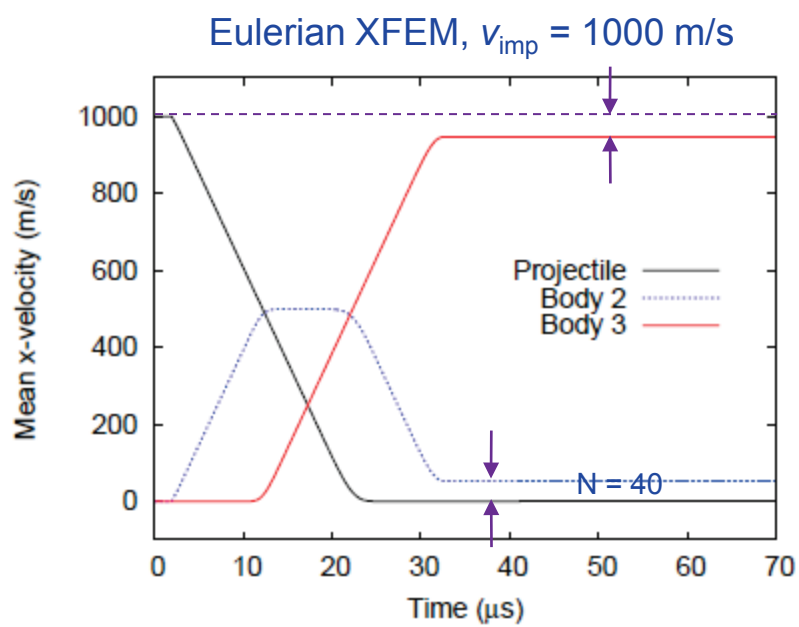
With XFEM, the material interfaces are no longer subgrid.

- Momentum transfer does not begin before full gap closure during impact.
- Gap opening allows recoil to happen naturally.



For v_{imp} = 10 m/s and 1000 m/s, XFEM captures the expected behavior:





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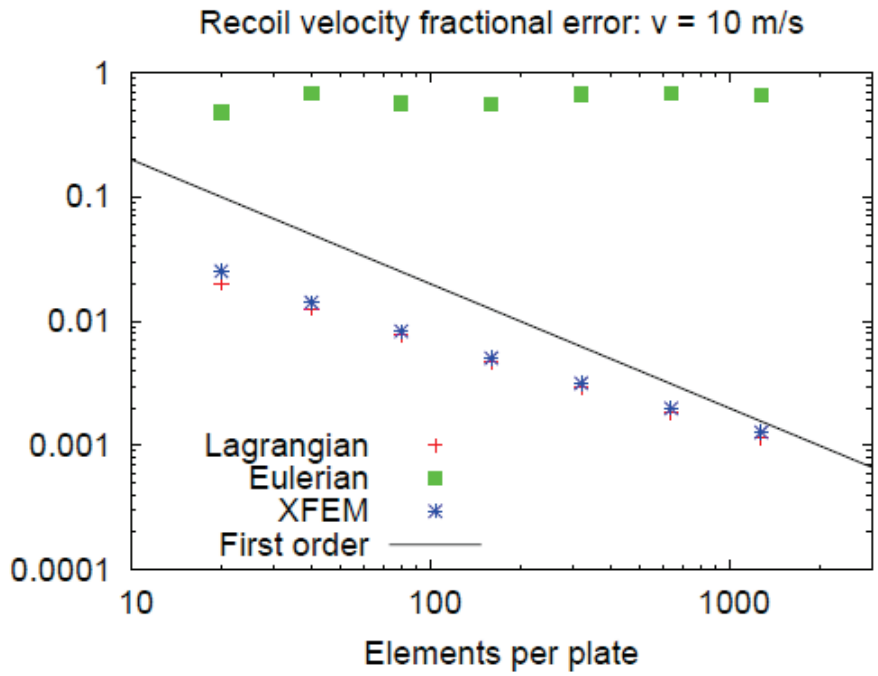
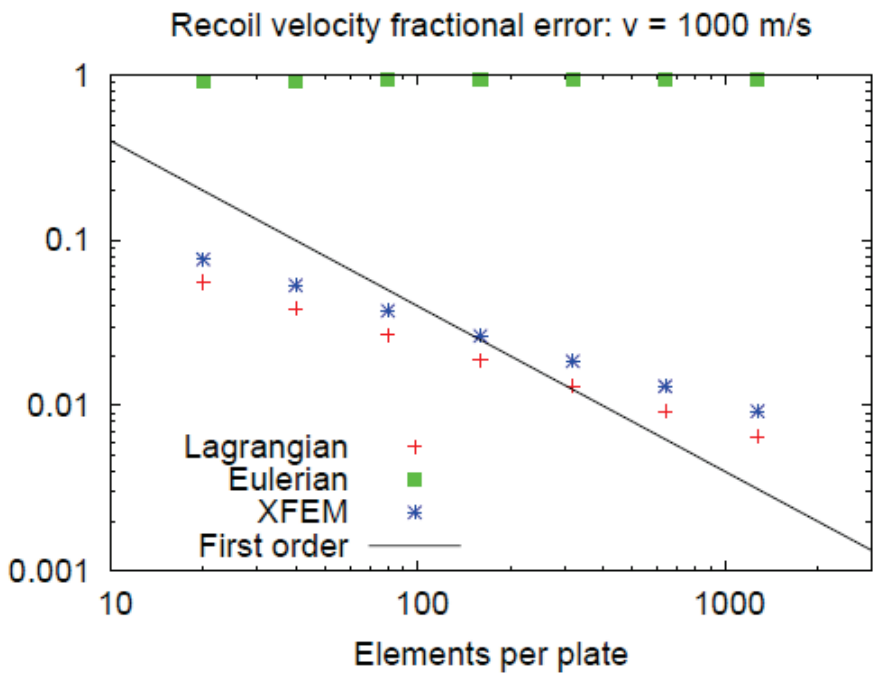
Error convergence for recoil velocity is only observed with XFEM

Error convergence under spatial refinement shows XFEM provides accuracy near that of the Lagrangian formulation. The standard Eulerian formulation fails to converge.





Mean convergence rate

	v_{imp} = 10 m/s	v_{imp} = 1000 m/s
Lagrangian	0.688	0.518
Eulerian	-0.079	0.007
XFEM	**0.718**	**0.509**

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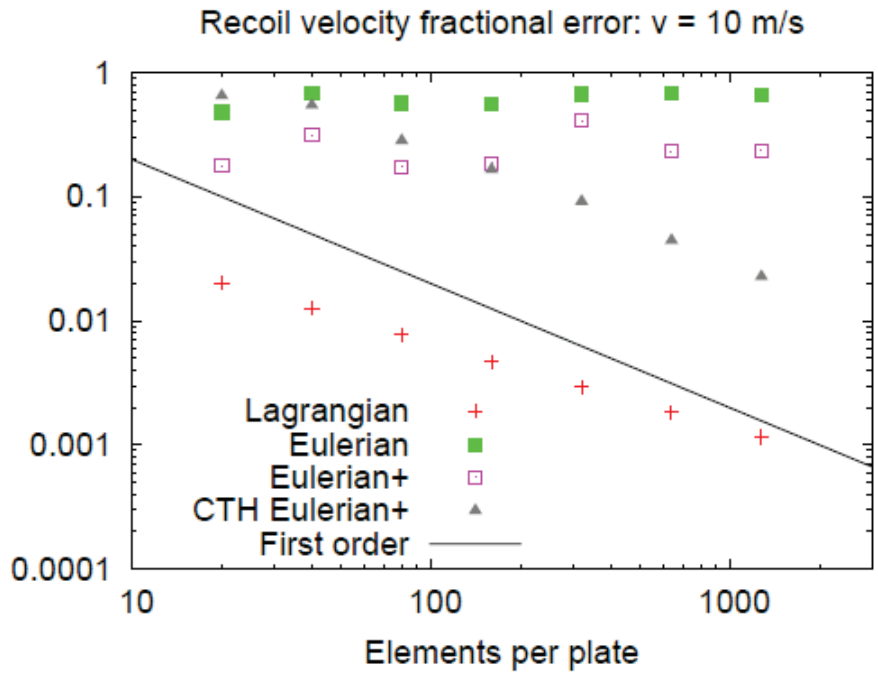
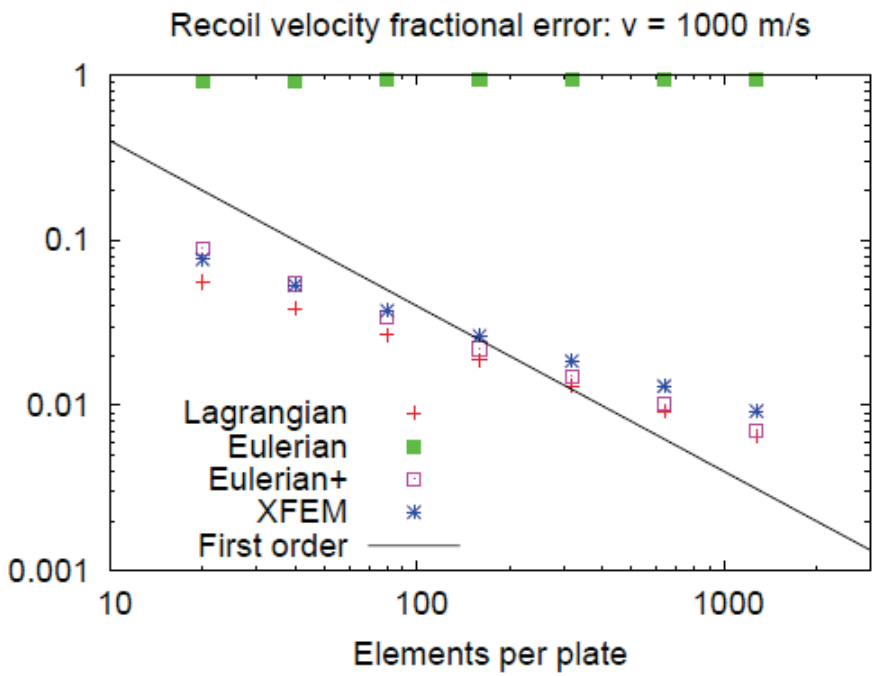
Ad hoc modifications to standard Eulerian FEM give mixed results

Error convergence under spatial refinement shows XFEM provides accuracy near that of the Lagrangian formulation. The standard Eulerian formulation fails to converge.





Mean convergence rate

	v_{imp} = 10 m/s	v_{imp} = 1000 m/s
Lagrangian	0.688	0.518
Eulerian+	-0.070	0.588
XFEM	**0.718**	**0.509**
CTH Eulerian+	0.807	---

Eulerian+:
Eulerian with "intermaterial fracture" (imposed interfacial material damage)

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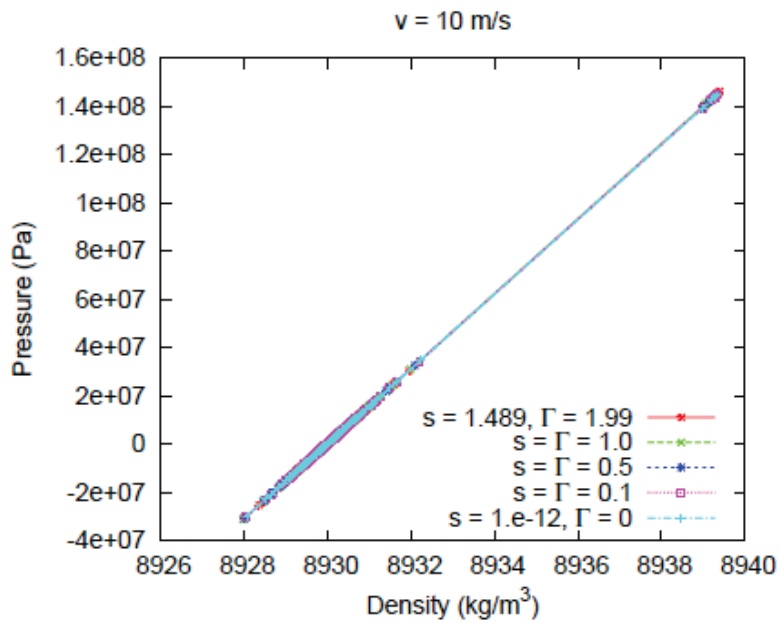
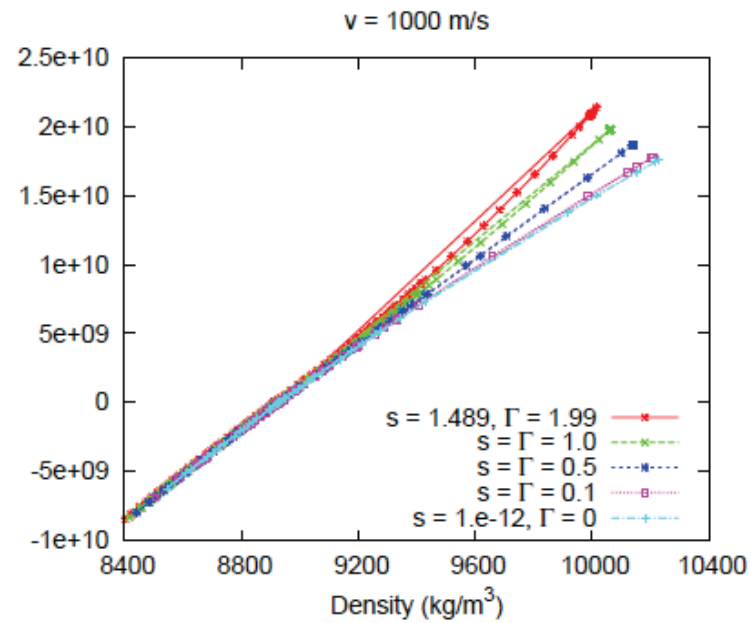
Material response can introduce dissipation, leading to analysis complications

At high impact velocity, the analysis shown here is ambiguous:

- Pressure-density response is not linear.
- The response is not irreversible, even with $s = 0$ and $\Gamma = 0$.
- Artificial viscosity cannot be neglected.





Further, more generality is needed: will convergence behavior hold when dissipationless, symmetric assumptions are abandoned? Alternate analysis for reference solution is needed.

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Solutions can also be obtained using the method of characteristics

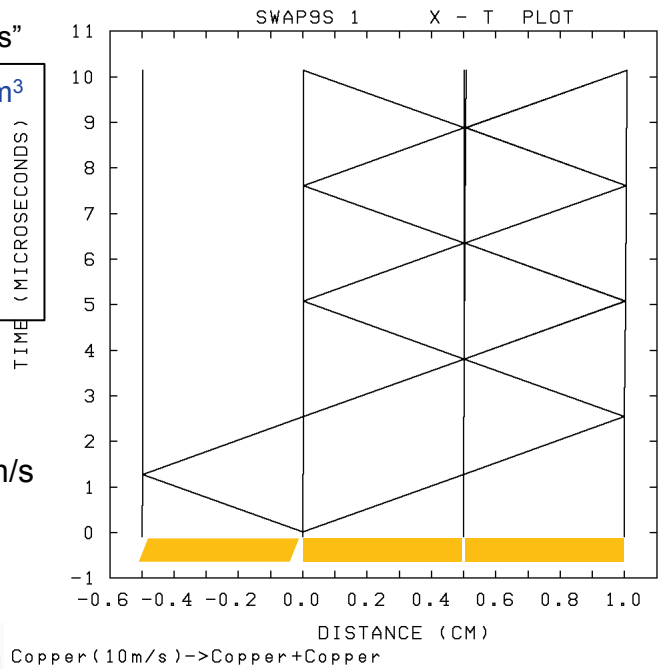
A more general approach yields solutions to problems in Eulerian one-dimensional solid dynamics by formulating the Euler equations as characteristic equations and applying a method-of-characteristics (MOC) approach.

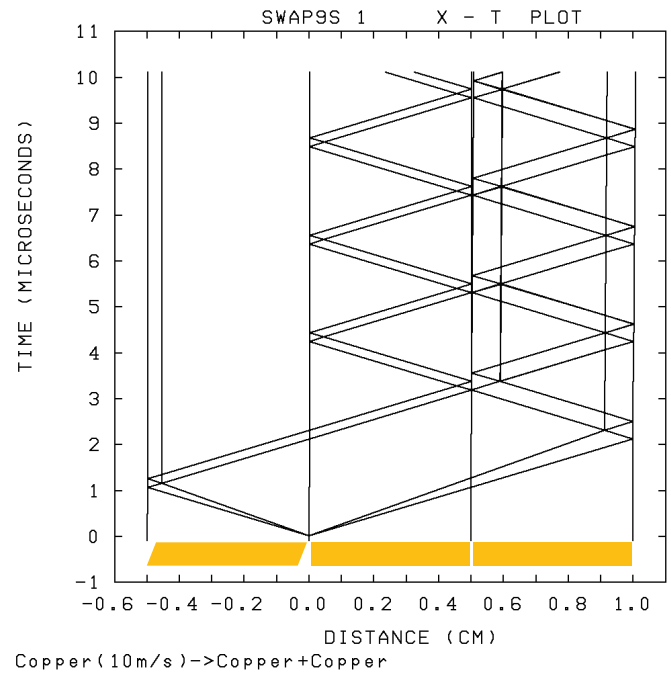
→ MOC solution is second-order in the strain increment.
→ Sandia's MOC code, SWAP-9 (Shock Wave Analysis Program*) computes solutions in one dimension for arbitrary EOS and constitutive models.

"Dissipationless"

ρ_0 = 8930 kg/m^3
c_0 = 3940 m/s
$E = \rho_0 c_0^2$
$\Gamma_0 = 10^{-4}$
s = 0.5
ν = 0.0

v_r = 10.000 m/s





"Real"

ρ_0 = 8930 kg/m^3
c_0 = 3940 m/s
$E = \rho_0 c_0^2$
Γ_0 = 1.99
s = 1.489
ν = 0.355
Y = 89.7 MPa

v_r = 9.873 m/s

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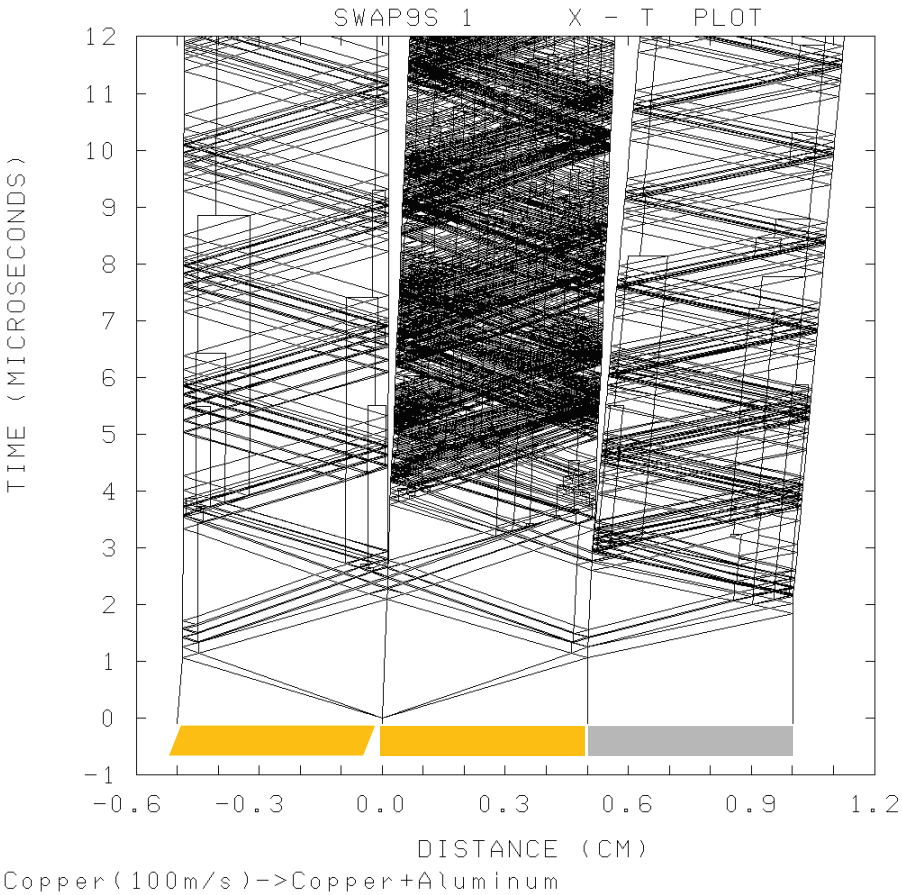
* Barker and Young, Sandia technical report SLA-74-0009,

MOC solution allows the problem to be extended to multiple materials with real EOS

SWAP-9* allows reference solutions to be computed for material representations that include a Mie-Grüneisen EOS and an elastic-perfectly-plastic yield model.



Copper plate impact on cohesionless copper + aluminum target

Plate thickness: τ = 0.5 cm

Copper

ρ_0 = 8930 kg/m^3
c_0 = 3940 m/s
$E = \rho_0 c_0^2$
Γ_0 = 1.99
s = 1.489
ν = 0.355
Y = 89.7 MPa

Aluminum

ρ_0 = 2707 kg/m^3
c_0 = 5250 m/s
$E = \rho_0 c_0^2$
Γ_0 = 1.97
s = 1.370
ν = 0.334
Y = 265.0 MPa

v_{imp} = 10 m/s → v_r = 14.004 m/s
v_{imp} = 100 m/s → v_r = 127.22 m/s
v_{imp} = 1000 m/s → v_r = 1357.0 m/s

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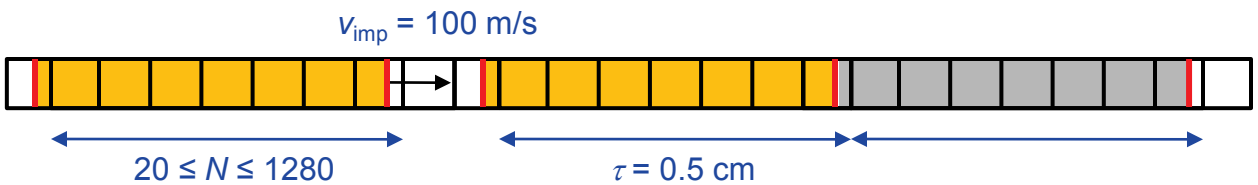
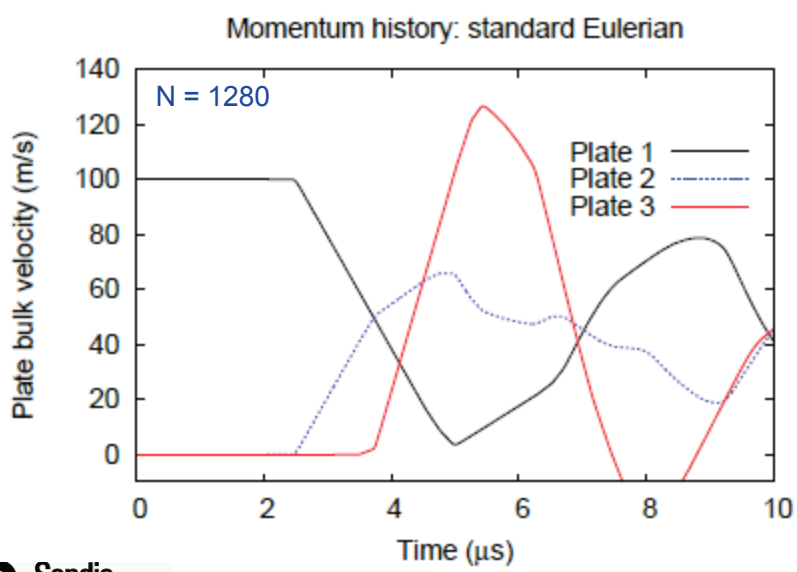
* Barker and Young, Sandia technical report SLA-74-0009,

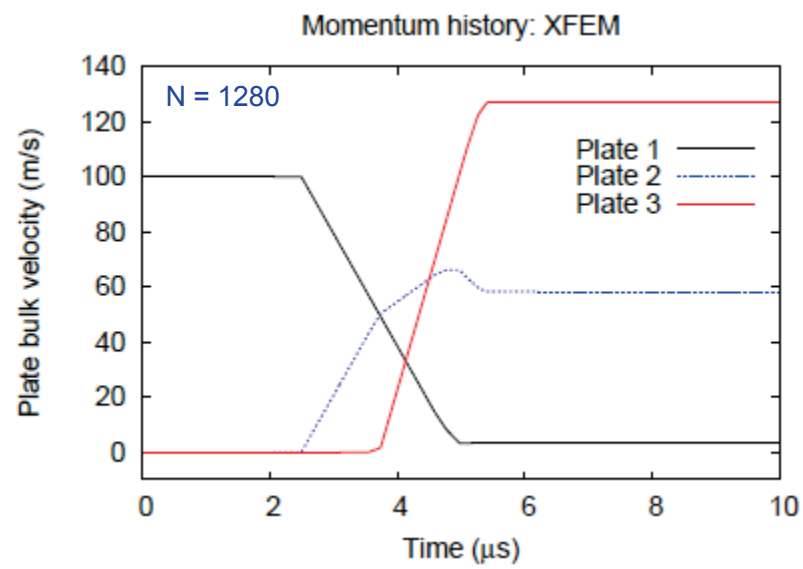
XFEM captures asymmetric recoil in two-material cohesionless impact

The cohesionless impact problem with copper projectile plate and copper + aluminum target is set up in ALEGRA using Lagrangian, Eulerian, and XFEM configurations.



For v_{imp} = 100 m/s, XFEM recovers correct behavior; standard FEM requires damage.





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XFEM captures asymmetric recoil in two-material cohesionless impact

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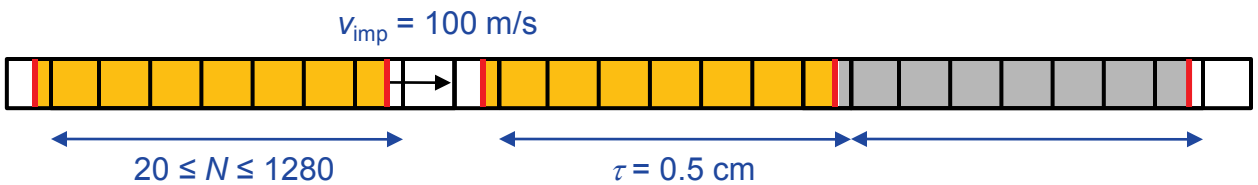

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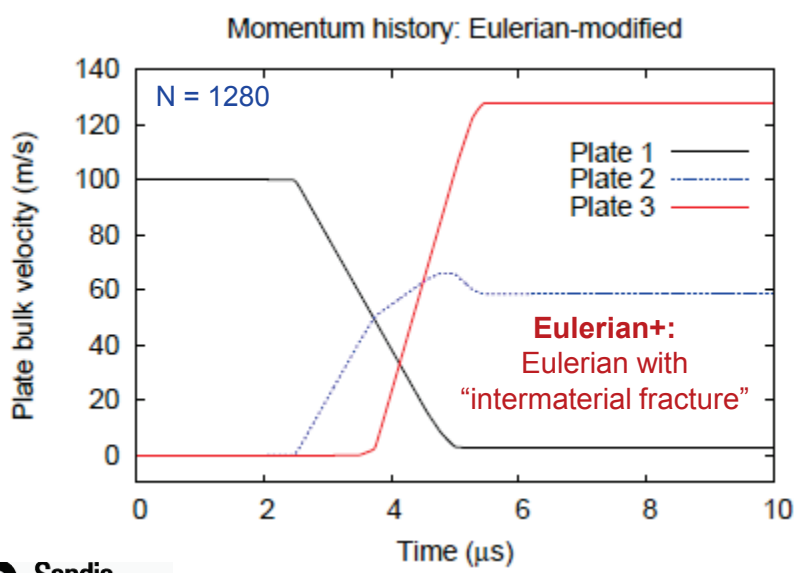

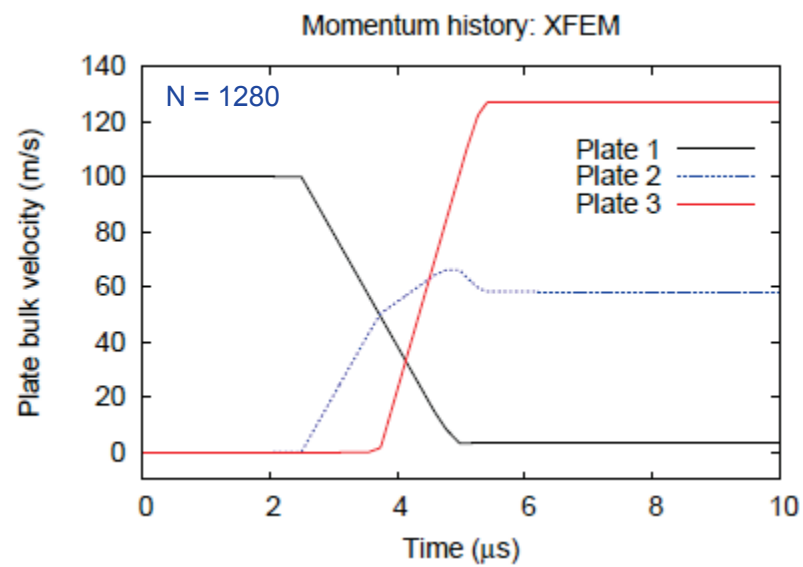

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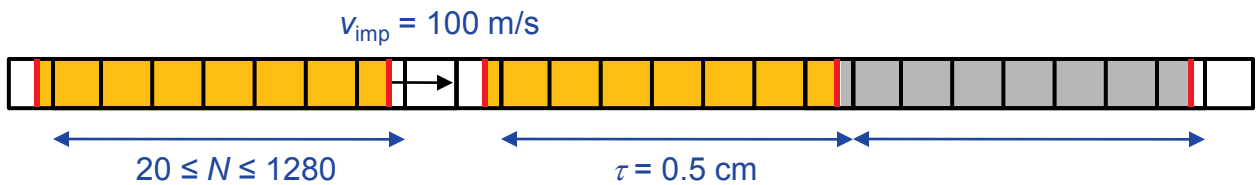
Standard FEM with modifications also captures asymmetric recoil

The cohesionless impact problem with copper projectile plate and copper + aluminum target is set up in ALEGRA using Lagrangian, Eulerian, and XFEM configurations.



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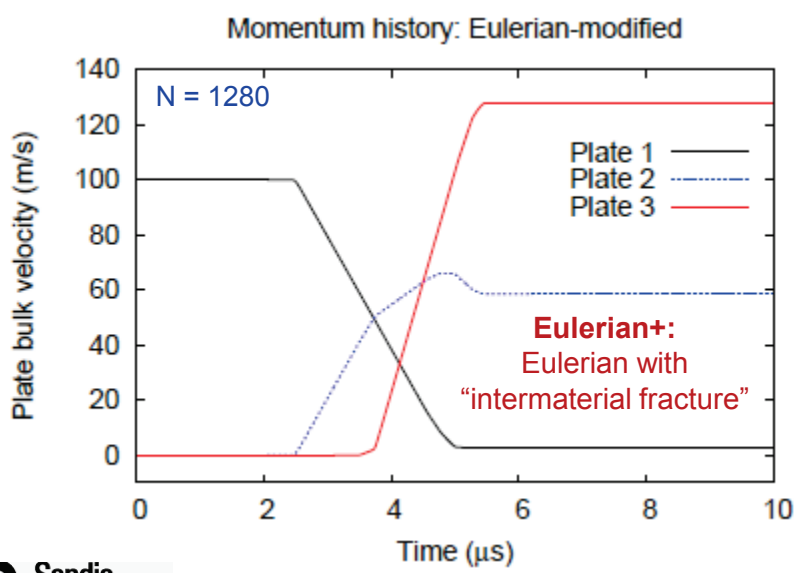

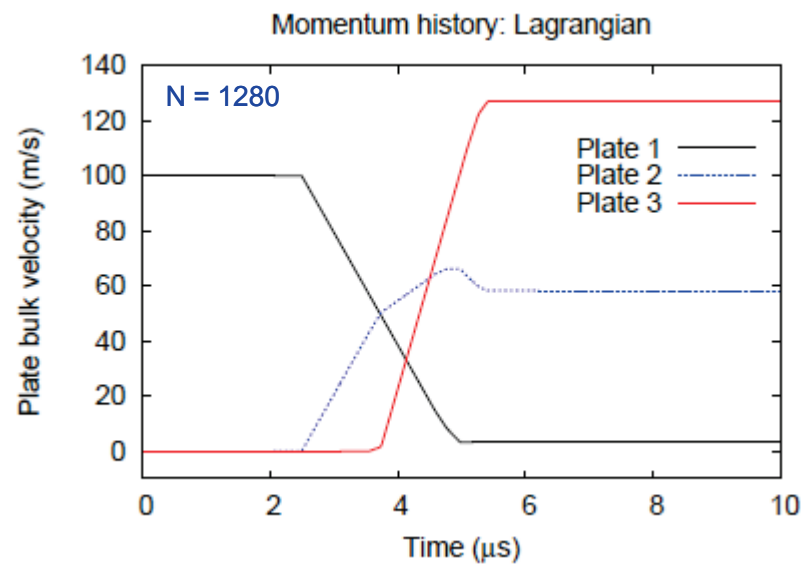

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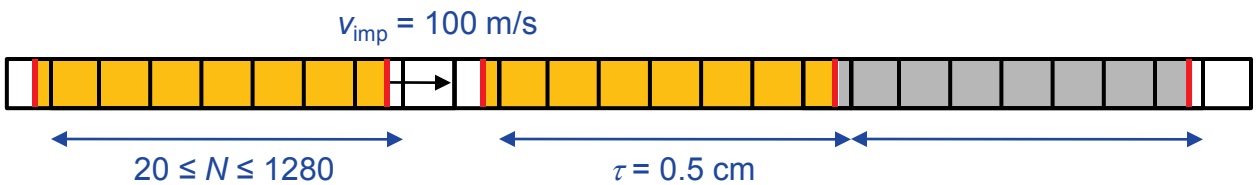
Verification analysis proves that XFEM converges to the MOC solution

The cohesionless impact problem with copper projectile plate and copper + aluminum target is set up in ALEGRA using Lagrangian, Eulerian, and XFEM configurations.



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Fractional error:
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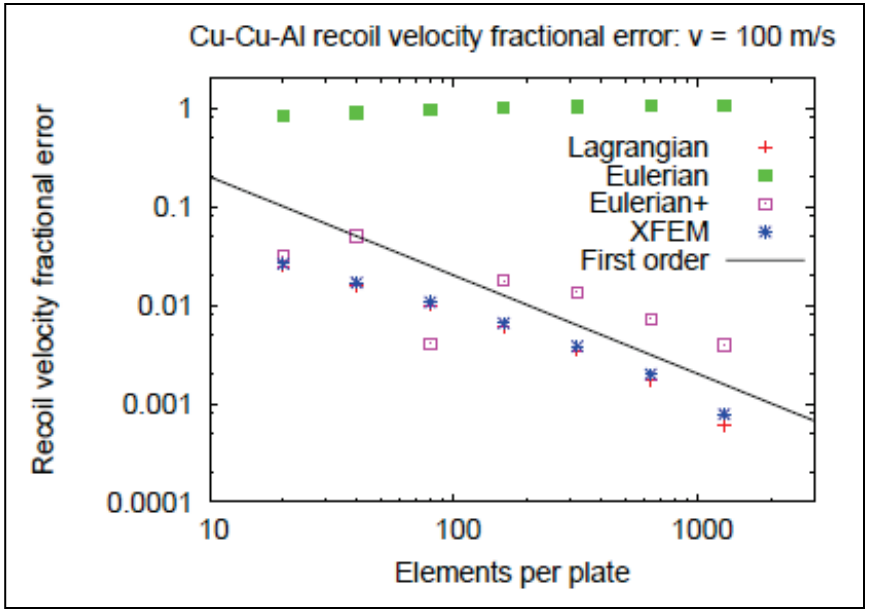

- Lagrangian: standard unenriched ALEGRA, no remap.
- Eulerian: standard unenriched ALEGRA with remap.
- Eulerian+: activate internal *ad hoc* interface physics modifications.

<u>Mean convergence rates</u>

Lagrangian:	0.89
Eulerian:	-0.05
Eulerian+:	0.50
XFEM:	0.85

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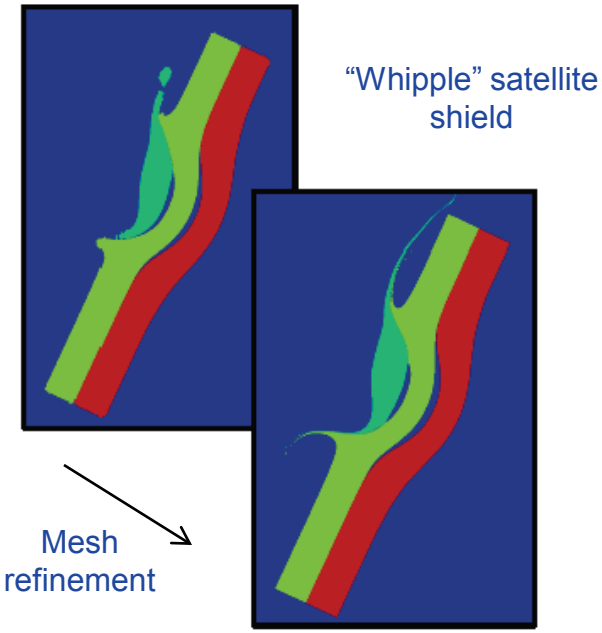
Summary and conclusions

- Verification analysis was performed using Lagrangian-frame, Eulerian-frame and XFEM simulations, for a 1D “Newton’s cradle” (cohesionless impact) problem.
- Exact solutions were computed for idealized problem. Approximate reference solutions were computed using MOC for real problem.
- In both cases, XFEM recovered the Lagrangian behavior and converged to the exact or reference solution with expected accuracy.

→ XFEM shows tremendous promise for multimaterial shock-hydro problems.

→ Verification testing and initial beta testing are both nearly complete.



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EXTRA SLIDES

XFEM computational cost is significant

Use of XFEM in ALEGRA is a compromise between computational cost and accuracy. Dramatically increased accuracy of XFEM requires roughly an order-of-magnitude increase in computational cost:

Total elapsed run time for Cu-Cu-Al, 100-m/s cohesionless impact simulation with N = 1280 elements per plate…

Lagrangian:	1192 s (serial, 3840 elements)
Eulerian:	1193 s, (4 cores, 5312 elements)
Eulerian+:	1026 s (4 cores, 5312 elements)
XFEM:	10086 s (8 cores, 5312 elements)

However, it should be noted that significantly lower resolution is required to capture important interfacial effects in XFEM compared to standard Eulerian FEM.

This implementation of XFEM in ALEGRA is revolutionary and needs rigorous verification

To characterize accuracy and correctness of XFEM implementation in ALEGRA for shock problems, a shock verification problem is needed.

Crack propagation
Hansbo and Hansbo, *CMAME* 2004

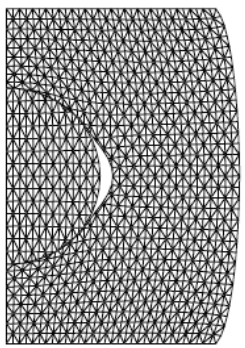


Fig. 4. Displacements for the contact problem.

Circular void expansion
Dolbow *et al*, *CMAME* 2008

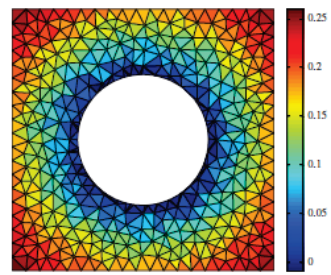
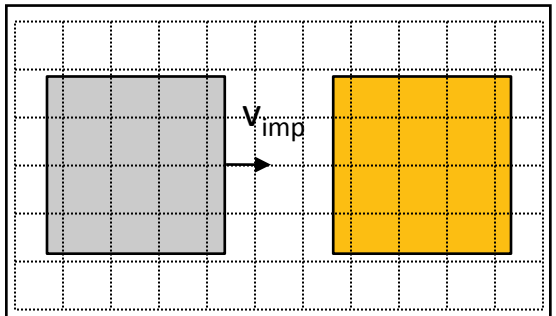


Fig. 8. Numerical approximation u^h to the solution u of the one-sided problem.

1D high-velocity impact and shock
Present work



See Carpenter *et al.*, *IJNME* 1991

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Some restrictive assumptions are necessary to maintain dissipationless character

To obtain this very simple solution, what material response must be assumed?

- Hugoniot is linear → linearized Mie-Grüneisen EOS may be used.

$$U_s = c_0 + sU_p$$

$$p(\rho, E) = p_H + \Gamma_0 \rho_0 [E - E_H(\rho)]$$

- Wave speed is a fixed material property → compression and release waves have the same fixed speed.

$$s = 0$$

Copper: s = 1.489

- Thermal contribution to Mie-Grüneisen is negligible → only volumetric (Hugoniot) contribution affects the pressure.

$$\Gamma_0 = 0$$

Copper: Γ_0 = 1.99

- Constitutive response is purely elastic

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An exact solution for Newton's cradle can also be obtained analytically

Three assumptions are necessary to make the problem dissipationless:

1. Deformation is uniaxial → stress tensor σ is spherical and $p = \mathrm{tr}(\sigma)/3$
2. Material response is elastic → plasticity and fracture play no role
3. Thermal contribution to pressure much smaller than volumetric contribution

Pressure in the initial impact event is then determined solely by the Hugoniot:

$$\begin{aligned} p_{1,1} &= p_{1,0} + \rho_{1,0}(U_{1,1} - u_{1,0})(u_{1,1} - u_{1,0}) \\ p_{2,1} &= p_{2,0} + \rho_{2,0}(U_{2,1} - u_{2,0})(u_{2,1} - u_{2,0}) \end{aligned}$$

An additional assumption is required: shock wave speed = sound wave speed: fixed.

$$\begin{aligned} U_{1,1} &= u_{1,1} - c_1 \\ U_{2,1} &= u_{2,1} + c_2 \end{aligned}$$

Setting impact pressures and material velocities to be equal and solving, we obtain the shock state (j = 1) in imp

$$\rho_{1,1} = \rho_{1,0}\frac{U_{1,1} - u_{1,0}}{U_{1,1} - u_{1,1}}$$

$$u_{2,1} = \frac{u_{1,0}}{2} = \frac{v_{imp}}{2}$$

$$\rho_{2,1} = \rho_{2,0}\frac{U_{2,1} - u_{2,0}}{U_{2,1} - u_{2,1}}.$$

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An exact solution for Newton's cradle can also be obtained analytically

Pure transmission across the plate-2/plate-3 interface implies plate 3 has the same shock state as plate 2.

Our assumptions imply the release state also lies on the Hugoniot. Same expressions and process then yield plate-3 release state (j = 2), when pressure is set equal to zero:

$$p_{3,2} = p_{3,1} + \rho_{3,1}(U_{3,2} - u_{3,1})(u_{3,2} - u_{3,1}) = 0.$$

Substituting Hugoniot pressure, density, and particle speed for the shock state in plate 3 into this expression, and solving for plate-3 release particle speed yields:

$$u_{3,2} = u_{3,1} + u_{2,1} = 2u_{2,1}$$

$$\boxed{u_{3,2} = u_{1,0} = v_{imp}.}$$

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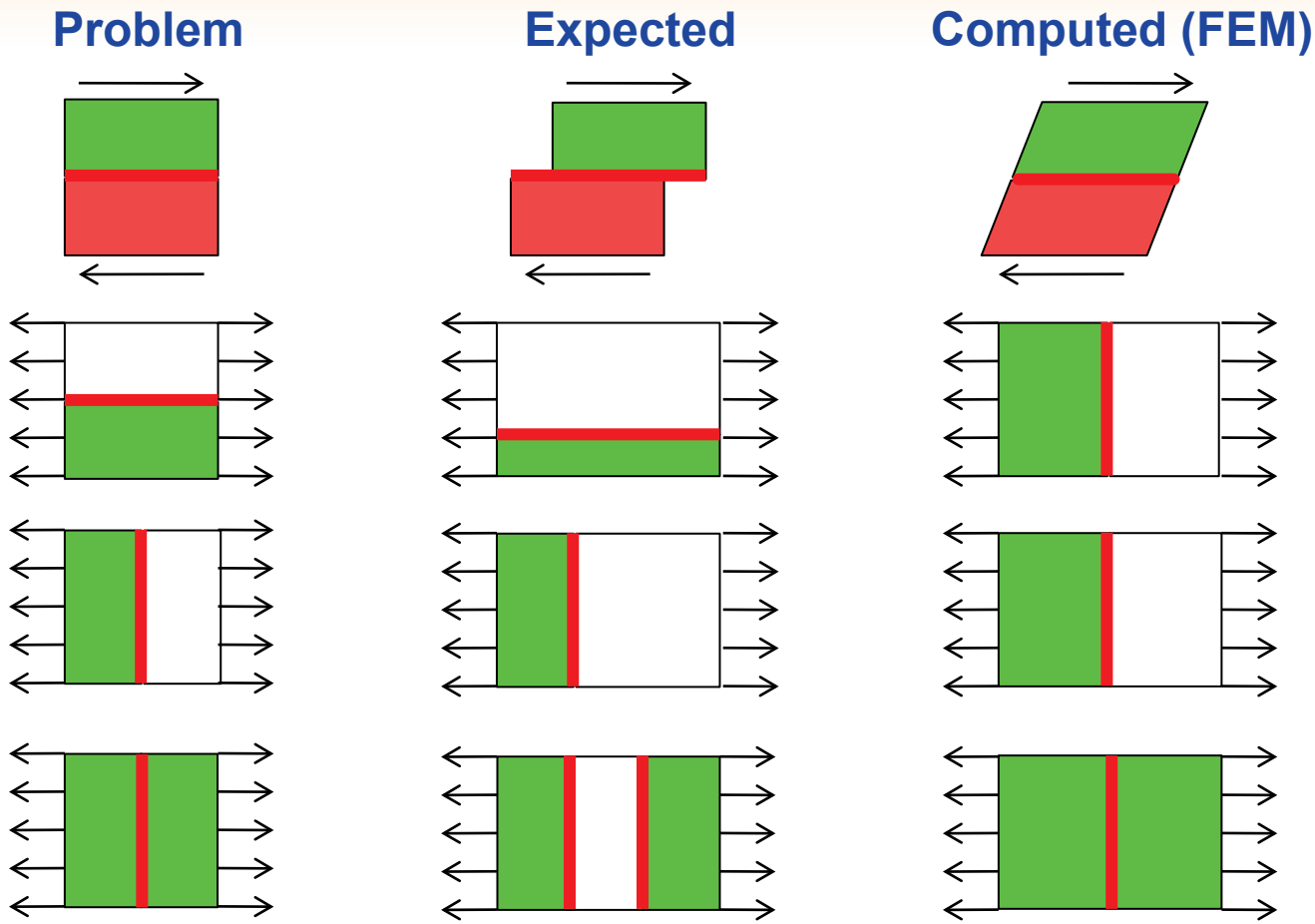
Dynamical fallacies are inherent to intra-element interfaces in standard FEM



- Erroneous material states induced by slipping and expansion
- Inadmissibility of void-opening and relative motion between materials
- Excessive adhesion and erroneous dissipative effects

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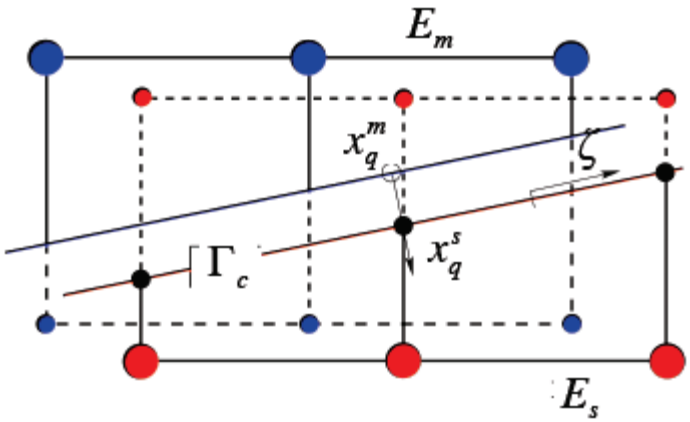
XFEM eliminates these dynamical fallacies

With XFEM, the issues surrounding subgrid multimaterial physics in the Eulerian frame are addressed directly:

- Advection employs a monotonicity-preserving Intersection Remap method with third-order limiting (J Mosso).
- Interfaces are captured using Pattern Interface Reconstruction, which reproduces linear interfaces in all orientations and has 2nd order accuracy for planar and curved surfaces (J Mosso).
- Forces on enriched-element nodes lying on interfaces are controlled by a forward-increment Lagrange multiplier contact algorithm (Tom Voth).



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