Arctic Sea Ice Mechanics with an Elastic-Decohesive Constitutive Model

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Sandia National Labs
Applied Mathematics and Applications

IPAM Mathematics of Sea Ice Workshop
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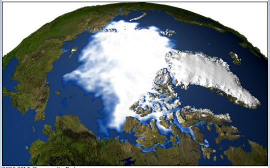
Acknowledgements

- Prof. Deborah Sulsky, UNM
- Prof. Howard Schreyer, UNM
- Dr. Pavel Bochev, SNL
- Funding from LDRD, NNSA BER, NSF(UNM)

1979 SSMI Composite Data



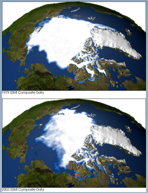
2003 SSMI Composite Data

earthobservatory.nasa.gov

Outline

Introduction

- Arctic sea ice plays important role in global climate by reflecting solar radiation and insulating the ocean from the atmosphere
- Due to feedback effects, the Arctic sea ice cover is changing rapidly
- Accurate, high-resolution models must incorporate
  - annual cycle of growth and melt due to radiative forcing
  - mechanical deformation due to surface winds, ocean currents, and Coriolis forces
  - localized effects of leads and ridges



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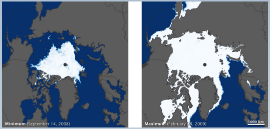

earthobservatory.nasa.gov


alaska.usgs.gov

Importance of Leads

- In winter heat transfer between ocean and atmosphere occurs primarily in leads
- Where new ice production (and brine flux to ocean) occurs in winter
- Satellite data show deformation focused into large linear features associated with leads

www.nsidc.org

www.arctic.noaa.gov

Sea Ice Model Components

2-D momentum equation for ice velocity $\mathbf{v}$

$$\rho\bar{h}\frac{d\mathbf{v}}{dt} = \nabla\cdot(\bar{h}\boldsymbol{\sigma})+\mathbf{t}_a+\mathbf{t}_w-\mathbf{f}_c-\rho\bar{h}g\nabla H$$

$$\mathbf{f}_c = 2\rho\bar{h}\omega\sin\phi(e_3\times\mathbf{v})$$

$$\mathbf{t}_a = C_a\|\mathbf{v}_a\|\mathbf{v}_a$$

$$\mathbf{t}_w = C_w\|\mathbf{v}-\mathbf{v}_w\|(\mathbf{v}-\mathbf{v}_w)$$

Ice thickness distribution for variations in thickness (h)

$$\frac{dg}{dt} + (\nabla\cdot\mathbf{v})g + \frac{\partial(fg)}{\partial h} = \psi$$

$$\bar{h} = \int_0^\infty hgdh$$

(Thorndike *et al., 1975, JGR, Vol. 80, No. 33, 4501-4513*)

1-D heat equation for temperature (T) and changes in thickness $(f = \partial h/\partial t)$

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \kappa I_0 \mathrm{e}^{-\kappa z}$$

$$F_w - k\frac{\partial T}{\partial z} = -q\frac{\partial h}{\partial t} \qquad F_a + k\frac{\partial T}{\partial z} = -q\frac{\partial h}{\partial t}$$

(Bitz and Lipscomb, 1999, JGR, Vol. 104, C7, 15669-15677) (Maykut and Untersteiner, 1971, JGR, Vol. 76, No. 6, 1550-1575)

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Viscous-Plastic Constitutive Model

Constitutive Relation

$$\mathbf{N} = \bar{h}\boldsymbol{\sigma} = 2\eta\dot{\varepsilon} + (\zeta - \eta)\mathrm{tr}(\dot{\varepsilon})\mathbf{I} + \frac{P\mathbf{I}}{2}$$

$$\zeta = \frac{P}{2\Delta}$$

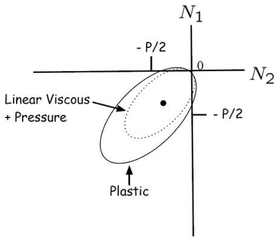
$$\eta = \frac{\zeta}{e^2} = \frac{P}{2\Delta e^2}$$

$$\Delta = \Big((\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2)(1 + e^{-2}) + 4\dot{\varepsilon}_{12}^2 e^{-2} + 2\dot{\varepsilon}_{11}\dot{\varepsilon}_{22}(1 + e^{-2})\Big)^{1/2}$$

(Hibler, 1979, J. Phys. Oceanography, Vol.9, 815-845)

$$\boxed{\rho\bar{h}\frac{d\mathbf{v}}{dt} = \nabla\cdot(\bar{h}\boldsymbol{\sigma}) + \mathbf{t}_a + \mathbf{t}_w - \mathbf{f}_c - \rho\bar{h}g\nabla H}$$

Yield Curve

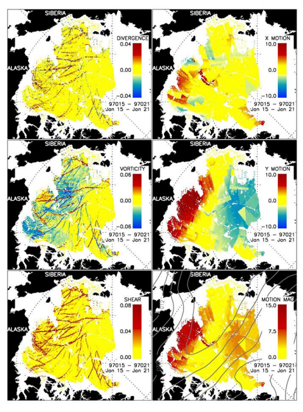


Ice Strength

$$P = P^*\bar{h}\mathrm{e}^{-C(1-A)}$$

Motivation for New Model

- Isotropic rheology not appropriate for high-resolutions where a single lead can dominate deformation



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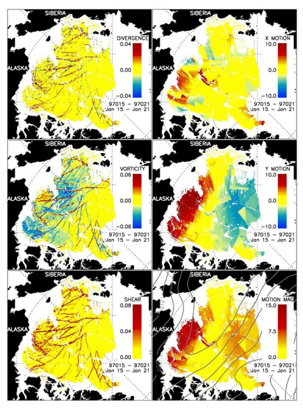
  → elastic-decohesive rheology



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Motivation for New Model

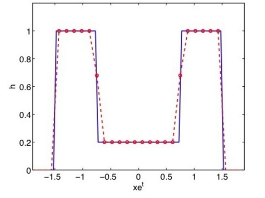
- Artificial diffusion in transport can lead to errors in thickness and smearing of ice edge

Horizontal transport

$$\frac{da}{dt} = -(\nabla \cdot \mathbf{v})a$$

$$\frac{dv}{dt} = -(\nabla \cdot \mathbf{v})v$$

$$\mathbf{v} = (-\mathbf{x}, 0), h = v/a$$



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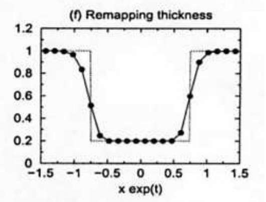

Lipscomb and Hunke, 2004, Mon. Weather Rev., 132(6), 1341-1354.

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  $\rightarrow$ material-point method

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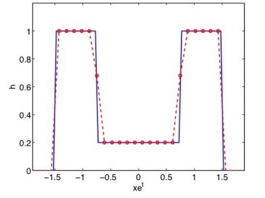

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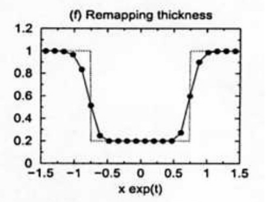

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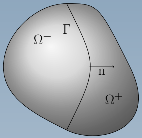
Elastic-Decohesive Constitutive Model

- Leads modeled as displacement discontinuities



Strong discontinuities

$$\mathbf{u} = \bar{\mathbf{u}} + [\![\mathbf{u}]\!] H_\Gamma$$

$$\boldsymbol{\varepsilon} = \nabla^s \mathbf{u} = \bar{\boldsymbol{\varepsilon}} + ([\![\mathbf{u}]\!] \otimes \mathbf{n})^s \delta_\Gamma$$

$$\boxed{\rho \bar{h} \frac{d\mathbf{v}}{dt} - \mathbf{f}_c = \nabla \cdot (\bar{h}\boldsymbol{\sigma}) + \mathbf{t}_a + \mathbf{t}_w}$$

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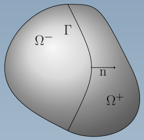

- Initiation and orientation of lead based on stress state

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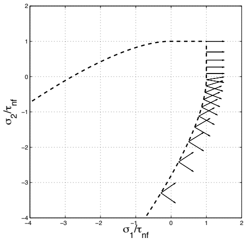
Damage function

$$\Phi(\boldsymbol{\tau}), \quad \boldsymbol{\tau} = \boldsymbol{\sigma}\mathbf{n}$$



Schreyer *et al.*, *2006, JGR 111, C11S26, doi:10.1029/2005JC003334*

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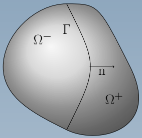

- Initiation and orientation of lead based on stress state
- Implemented like an elastic-plastic relation

Constitutive relation

$$\dot{\boldsymbol{\sigma}} = \mathbb{E} : (\dot{\boldsymbol{\varepsilon}} - ([\![\dot{\mathbf{u}}]\!] \otimes \mathbf{n}_\Gamma)^s \delta_\Gamma)$$

$$[\![\dot{\mathbf{u}}]\!] = \lambda \frac{\partial \Phi}{\partial \boldsymbol{\tau}}$$

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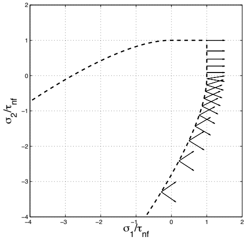

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Elastic-Decohesive Constitutive Model

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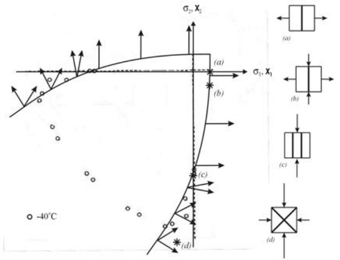
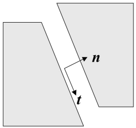
$$\Phi = \left(\frac{\tau_t}{\tau_{sm}}\right)^2 + \mathrm{e}^{\kappa B_n} - 1$$

$$B_n = \frac{\tau_n}{\tau_{nf}} - f_n \left(1 - \frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2}\right)$$



Schulson, 2001, Engng. Frac Mech., Vol. 68, 1879-1887



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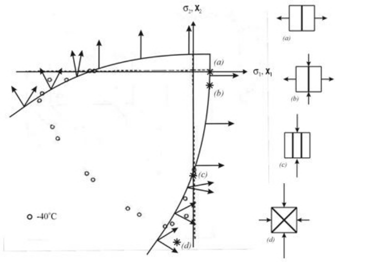

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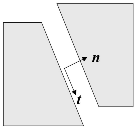

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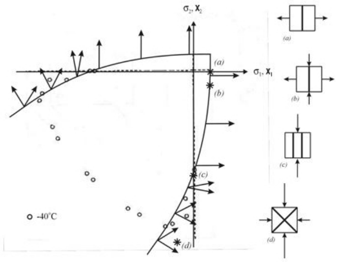

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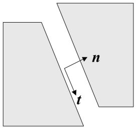

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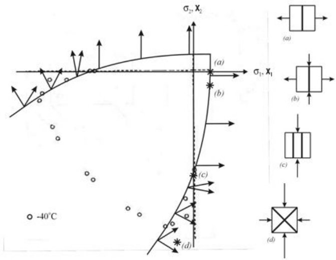
$\sigma_{tt} = \mathbf{t}\cdot\boldsymbol{\sigma}\cdot\mathbf{t}$, f_c compressive strength



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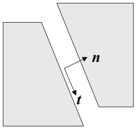

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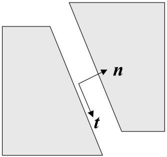
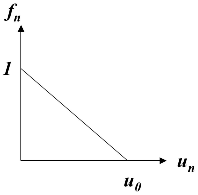
Softening function

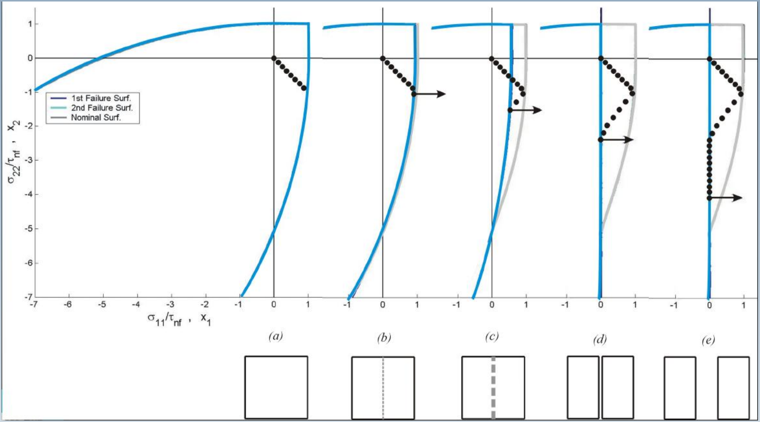
$$f_n = \left\langle 1 - \frac{u_n}{u_0} \right\rangle$$

$$[\![\mathbf{u}]\!] = u_n \mathbf{n} + u_t \mathbf{t}$$

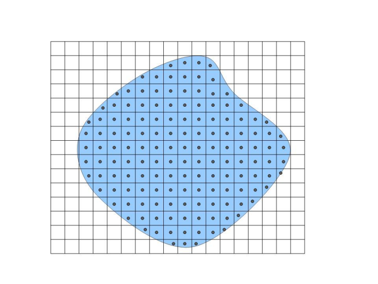




Failure Evolution



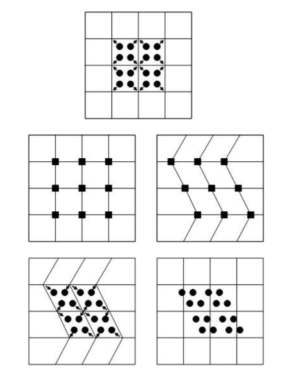
Material-Point Method (MPM)



- Domain divided into material points and background grid
- Lagrangian material points carry mass, momentum, thickness distribution, and internal variables for constitutive model
- Momentum equation solved on background grid
- Constitutive model, ITD, and thermodynamics evaluated at material point

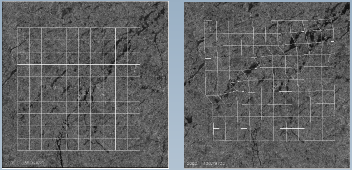
MPM Algorithm

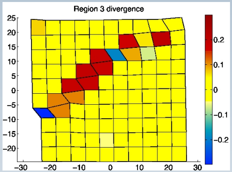
1. Map MP values to nodes
2. Calculate internal and external forces at nodes
3. Solve momentum equation on grid
4. Update MPs based on grid solution
5. Evaluate constitutive model at MPs
6. Update ITD based on mechan-ics/thermodynamics
7. Regrid



RADARSAT Geophysical Processor System (RGPS)

- Developed by Polar Remote Sensing Group at JPL
- Extracts sea ice motion data from SAR imagery using area and feature based tracking
- Points tracked can be interpreted as nodes of a grid
- Grid quantities such as divergence, shear, and vorticity can be derived





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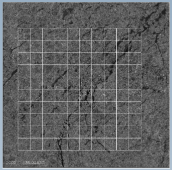
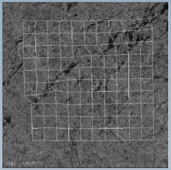
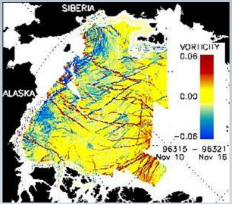




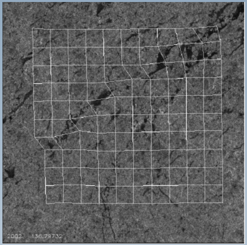


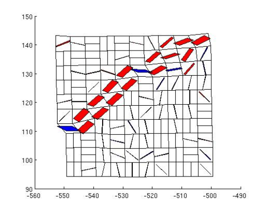
Decohesive Kinematics Using RGPS

- Want a procedure to extract information on cracks or leads from RGPS data
- Assume all deformation in cell due to discontinuity

$$\varepsilon = \frac{1}{L}(\llbracket \mathbf{u} \rrbracket \otimes \mathbf{n})^s$$

- Given ε from RGPS cell data, calculate best fit $\llbracket \mathbf{u} \rrbracket$ and $\mathbf{n}$





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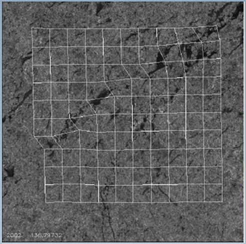

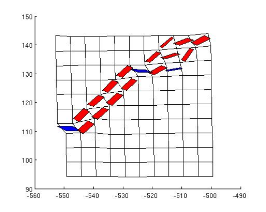

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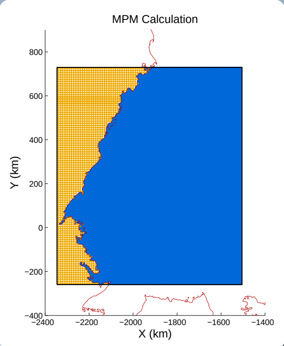
- 10 km square grid, 4 material points per cell initially
- Rigid material points for land boundary
- RGPS velocities used for boundary conditions



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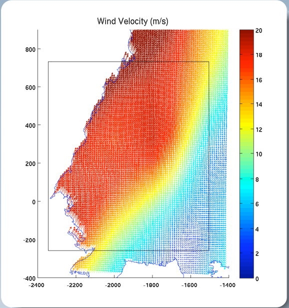
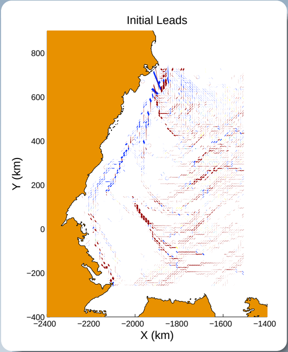
- Use RGPS data for 23 Feb - 11 Mar 2004 in Beaufort Sea region
- 10 km square grid, 4 material points per cell initially
- Rigid material points for land boundary
- RGPS velocities used for boundary conditions
- Ocean current and atmospheric wind forcing



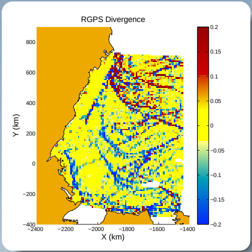
Beaufort Sea Calculation Setup

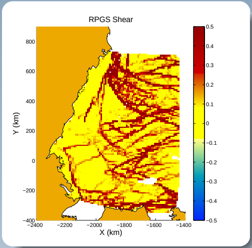
- Use RGPS data for 23 Feb - 11 Mar 2004 in Beaufort Sea region
- 10 km square grid, 4 material points per cell initially
- Rigid material points for land boundary
- RGPS velocities used for boundary conditions
- Ocean current and atmospheric wind forcing
- Initialize using kinematic crack data

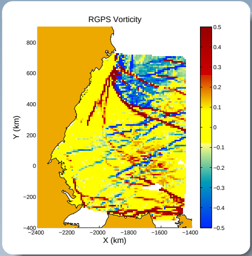


Beaufort Sea Net Deformation Results

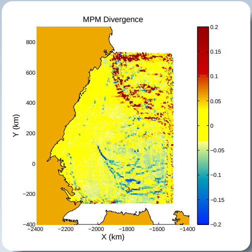
RGPS

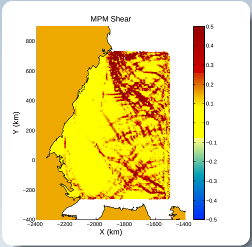


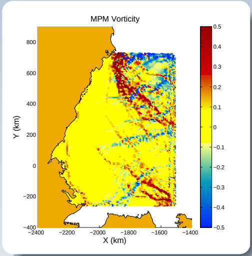




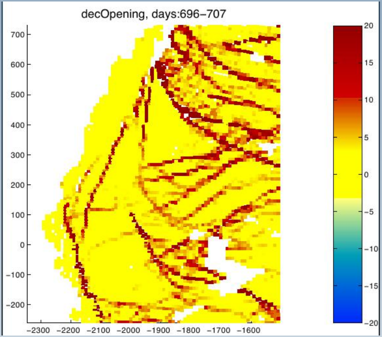
MPM



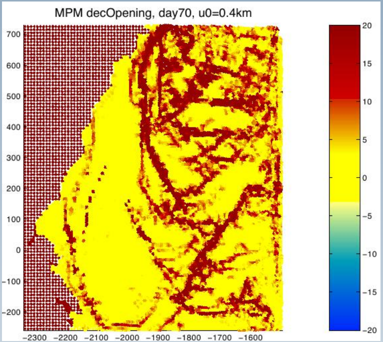




Beaufort Sea Results March 11 (Day 70)

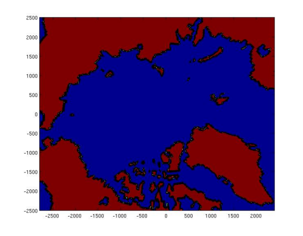


RGPS



MPM

Pan-Arctic Calculations Set-up



- 50 km resolution grid
- Rectangular - azimuthal equal area projection

- LANL CICE
  - Elastic-viscous-plastic rheology
  - Finite difference, linear remapping
  - Bitz and Lipscomb thermo
  - Five category ITD
- MPM Seaice
  - Elastic-decohesive rheology
  - Particle-in-cell
  - Bitz and Lipscomb thermo
  - Five category ITD

Forcing Data

- Atmosphere - Large and Yeager

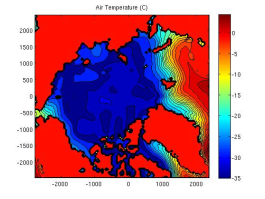
  (data1.gfdl.noaa.gov/nomads/forms/mom4/CORE.html)

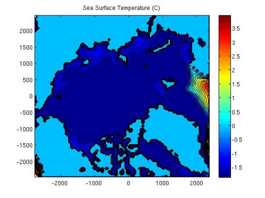
  - Air temperature
  - Specific humidity
  - Atmospheric wind
  - Downward short wave flux
  - Precipitation
- Ocean - PIOMAS

  (psc.apl.washington.edu/IDAO/data piomas.html)

  - Sea surface temperature
  - Sea surface salinity
  - Ocean currents
- Cloud fraction - AOMIP

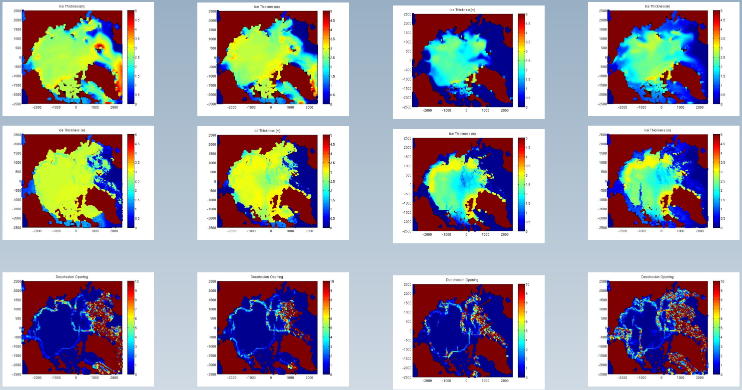
  (www.whoi.edu/page.do?pid=30580)





First Year Code Comparison

	March	June	September	December
CICE Thickness				
MPM Thickness				
MPM Opening				



Conclusions

- MPM sea ice model with anisotropic elastic-decohesive model
  - Initiation based on stress state
  - Orientation determined at initiation
  - Evolution of leads
  - Allows for predetermined areas of weakness
- Work in progess
  - Pan-Arctic calculations
  - More comparisons with data (metrics)

- Future work
  - Ridging and freezing algorithms tied to constitutive model
  - Curvilinear grid
  - Numerical improvements to model

alaska.usgs.gov