THE GRAFT TOOL: AN ALL-HEXAHEDRAL TRANSITION ALGORITHM FOR CREATING A MULTI-DIRECTIONAL SWEPT VOLUME MESH*

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ABSTRACT

Sweeping algorithms have become very mature and can create a semi-structured mesh on a large set of solids. However, these algorithms require that all linking surfaces be mappable or submappable. This restriction excludes solids with imprints or protrusions on the linking surfaces. The grafting algorithm allows these solids to be swept. It then locally modifies the position and connectivity of the nodes on the linking surfaces to align with the graft surfaces. Once a high-quality surface mesh is formed on the graft surface, it is swept along the branch creating a 2¾-D mesh.

Keywords: mesh generation, hexahedral meshing, refinement, sweeping, 2½-D

1. INTRODUCTION

Three-dimensional finite element analysis (FEA) is an important design tool for physicists and engineers. Before the analysis can begin, a mesh needs to be generated on the model. During the last several decades, much research has been devoted to mesh generation. Tetrahedral mesh generators are well developed and many have been implemented in software packages. Only recently has the research focus shifted to hexahedral meshes.

For most applications, hexahedral elements are preferred over tetrahedral elements for meshing 3-D solids [1,2]. Unfortunately, a high quality mesh of hexahedral elements is more difficult to generate. Minimally, the mesh needs to be conformal between adjoining solids and have high quality elements at the bounding surfaces. Because of the constraints on hexahedral elements, automatic generation of high quality hexahedral meshes on arbitrary 3-D solids has proven elusive [3].

Over the last several years much work has been put into sweeping algorithms. These algorithms can mesh a wide range of 2½-D (prismatic) solids. The sweeping algorithms generally take a 2-D unstructured quadrilateral mesh from the source surface and project it through the volume to the target surface. Sweeping algorithms have matured to handle non-planar, non-parallel source and target surfaces and variable cross-sectional area [4] as well as multiple source and target surfaces [5,6].

To maintain the structured mesh in the sweep direction, sweeping algorithms require the linking surfaces (those that connect the source to the target) to be mappable or submappable. This constraint limits the number of solids that

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can be meshed with these algorithms. They specifically exclude solids with imprints or protrusions on the linking surfaces. This paper introduces a new grafting algorithm that lifts this constraint on linking surfaces. The grafting algorithm locally modifies the structured mesh of the linking surfaces allowing the mesh to conform to additional surface features. Thus, the grafting algorithm can provide a transition between multiple sweep directions extending sweeping algorithms to 2¾-D solids.

After a brief definition of terms, the algorithm is presented with a simple example. The paper concludes with examples created using the algorithm.

2. DEFINITION OF TERMS

The grafting algorithm can be used for a single solid or for a set of connected solids that require conformal meshes between them. For ease of presentation, the 2¾-D solid discussed is assumed to be a single solid. The solids that generally benefit from the grafting algorithm have one major sweep direction with imprints or protrusions cluttering the linking surfaces. Figure 1 shows a sample 2¾-D solid.

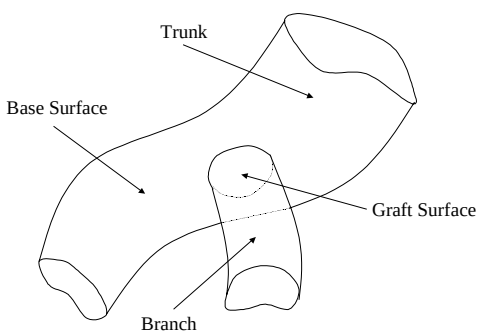

Figure 1. Definition of terms for a 2¾-D solid.

Usually the central and largest part of the solid is the major sweep direction and will be known hereafter as the *trunk*. The trunk often has protrusions from the linking surfaces that are sweepable subvolumes. These subvolumes are termed *branches*. The linking surface on the trunk that contains one or more branches is termed a *base surface* and begins with a structured mesh. The intersection of the trunk and branch is defined as a *graft surface*.

3. THE GRAFTING ALGORITHM

The goal of the grafting algorithm is to create a conformal mesh between the trunk and the branches composed of high quality elements. The algorithm has three major steps: meshing of the trunk, modification of the base surface mesh at the graft surface, and meshing of the branch. Each of these steps will be described in the following sections.

3.1 Meshing the Trunk

The first step in the grafting algorithm is to obtain a mesh on the trunk. The trunk is generally defined such that a structured meshing algorithm can create a successful mesh on it. In the examples used in this paper, the trunks are meshed with either volume mapping, submapping [7], or sweeping algorithms. It is not necessary, however, for the trunk to have a structured mesh on it. For severely complicated solids, it may not be easy to find a simple trunk. In these cases, it may be desirable to use an unstructured algorithm (e.g. Whisker Weaving [8], Plastering [9], or Hex-Tet [10]) to mesh the more complicated trunk and then use the grafting algorithm to create a transition to a more structured branch.

Regardless of the mesh scheme, special care must be taken when assigning the element size to the trunk. The mesh must be fine enough to resolve all small features of the trunk and the graft surface.

3.2 Creating the Graft

Once the trunk is meshed, the branches are grafted into the base surface mesh one at a time. Figure 2 shows a sample graft surface with the underlying structured mesh of the base surface.

The first step is to locate each graft surface on the mesh. The graft surface is separated into individual loops of curves that define the surface boundary. Each loop is temporarily meshed with a one-dimensional mesh that is twice as fine as the underlying mesh on the base surface. This is done to approximate the curve loop with a closed set of linear line segments.

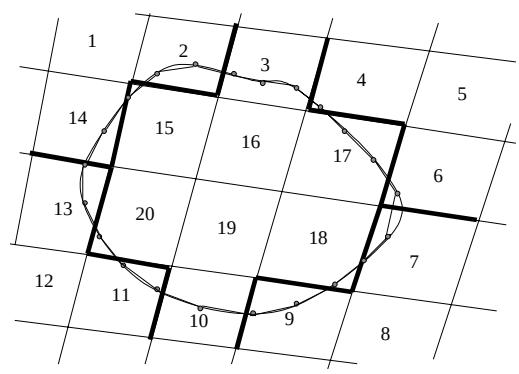

Figure 2. An example graft surface with underlying structured mesh from the base surface.

The mesh elements on the loop are traversed to find the set of mesh edges on the base surface that intersect the loop. A simple three-dimensional linear intersection routine is used to determine where each of these intersections occurs. In Figure 2, these intersected edges are highlighted with bold lines. Once the intersected edges of the base surface are located, the temporary mesh on the loop is deleted.

3.2.1 Smoothing the Mesh to the Loop

The next step is to adjust the surface mesh to conform to the loop. Using the set of intersecting mesh edges and the corresponding intersection locations, the closest node on each edge is moved to the loop. If two nodes are comparably close, the adjacent intersecting edges are checked to determine which node to move. The node that produces the highest quality quadrilaterals is moved. Figure 3 shows the modified surface mesh after the nodes are moved.

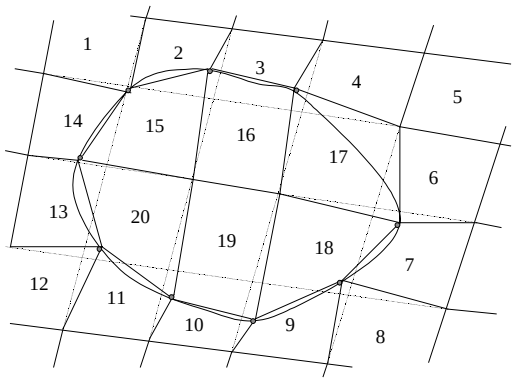

Figure 3. The nodes of the intersecting edges are moved to the loop.

By moving the nodes of the intersecting edges, occasionally the loop will be made to span diagonally across a quadrilateral. In Figure 3, element 17 is intersected diagonally by the loop. When this happens, a third node of the quadrilateral is moved to the loop. Again, which node to move is determined by the final mesh quality. Figure 4 shows the mesh that results from moving the nodes to the loop.

The final quality of the quadrilateral mesh inside the loop is limited by the resolution of the original mesh on the base surface and by the number of mesh faces that are diagonally intersected by the loop. Unfortunately, this smoothing procedure results in the poorest quality mesh faces at the loop boundary.

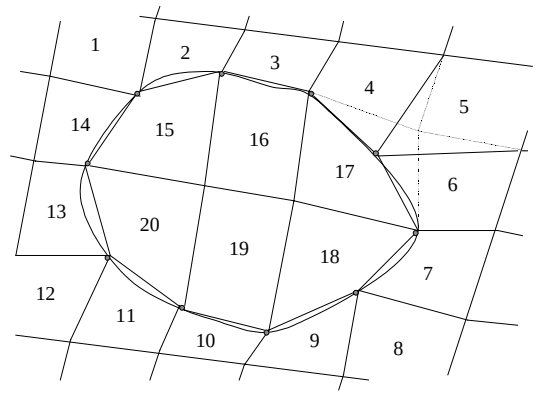

Figure 4. The base surface mesh is completely smoothed to the loop.

Another important consideration is the quality of the hexahedral mesh of the trunk immediately under the base surface. Generally, the further the surface nodes are moved, the poorer the quality of the underlying hexahedral mesh. In many cases, this smoothing process produces hexes with negative Jacobians. Though, the quality can be improved slightly by a smooth on the volume, the benefits in quality are not worth the computational expense of the smooth. Alone, smoothing is not sufficient to improve the mesh quality.

3.2.2 Refining Inside the Loop

As mentioned previously, it is important to have high quality elements near the surfaces of the solid. The quality of the elements at the surface of the branch is determined by the quality of the quadrilaterals immediately inside the bounding loops of the graft surface. In Figure 4, the poorest quality elements are at the bounding loop of the graft surface. To improve the quality of these elements, a refinement scheme is used that modifies the mesh connectivity locally.

The refinement scheme is best understood by inspection of the Spatial Twist Continuum (STC) [11], or dual of the mesh. A complete STC sheet is inserted directly inside the bounding loops of the graft surface. The sheet passes behind the first layer of hexes in the trunk creating a pillow of new hexes inside the loop [12]. Thus, the connectivity of the interior side of the hexes remains unchanged. This insures that the connectivity modification is local, especially on thin solids.

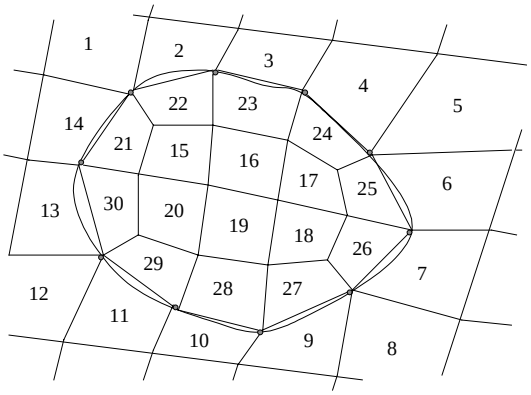

Figure 5. A pillow of elements is inserted directly inside the loops to improve element quality.

This process was applied to the mesh of Figure 4 and the resulting surface mesh is shown in Figure 5. Though only the surface mesh is shown, the new layer of hexes wraps around behind the existing hexes using the corner primitive suggested by Murdoch in [11]. The new layer of elements shows an improvement in quality and moves the lower quality elements to the center of the graft surface.

3.2.3 Improving Quality Outside the Loop

Before leaving the base surface, the elements immediately outside the loop are surveyed for poor quality. As mentioned above, some of these elements will have negative Jacobians due to the movement of the nodes. These quality issues can be corrected by inserting another STC sheet away from the loop. Finally, the mesh on the base surface is smoothed to optimize the node locations.

3.3 Meshing the Branches

When the quality of the mesh on the base surface has been improved, the branch is ready to be meshed. The set of quadrilateral elements inside the graft surface is defined as the source mesh for a sweeping algorithm. This mesh is then swept through to the end of the branch.

Previously it was mentioned that the trunk could be meshed with any volume mesh scheme, though most often a structured scheme was chosen. The same is true with the branches. Most often the branches are sweepable subvolumes with the graft surface as the source surface. However, any scheme, structured or unstructured, can be used to mesh the branches using the existing mesh on the graft surface.

4. EXAMPLES

Shown below are three simple examples of the grafting algorithm. Figure 6 shows a block-shaped trunk with a mapped volume mesh. The branch is a cylindrical protrusion from the trunk. The results of the grafting algorithm can be seen in Figure 7. The branch has been cut away to show the details of the base surface. Notice the high quality elements inside the graft surface. Further refinement was done outside the graft surface to improve the quality of the mesh.

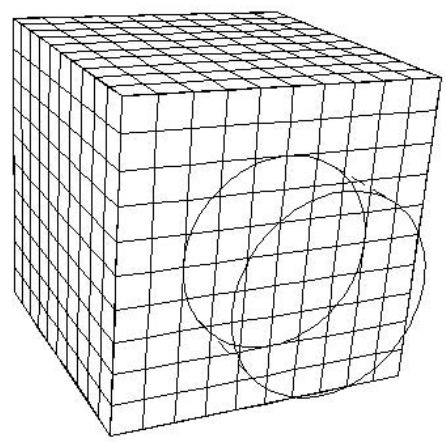

Figure 6. Mapped trunk mesh with cylinder branch before grafting.

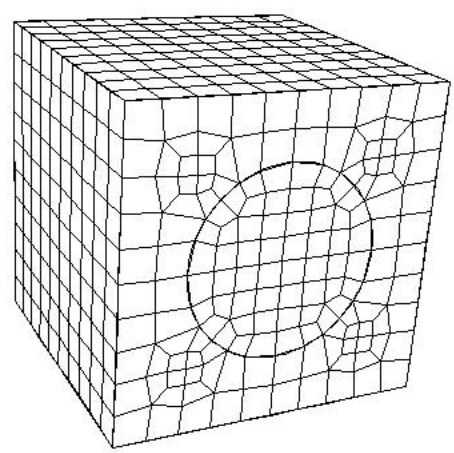

Figure 7. Cut away view of the base surface after grafting.

Figure 8 shows a slice from the center of the volume mesh from Figure 7. It is easy to see the layer of hexes that were added directly beneath the graft surface. Notice that the volume mesh is completely conformable. Additionally, all the hexes are of acceptable quality.

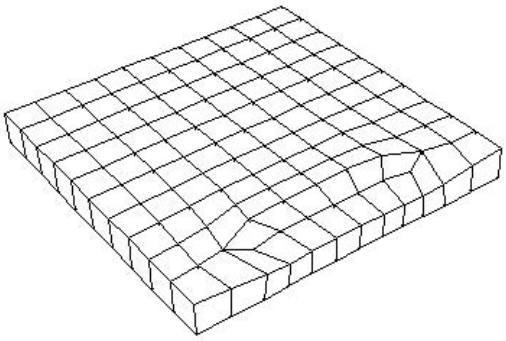

Figure 8. A slice of the volume of Figure 7 to show the internal hexes.

The trunk in Figure 9 was meshed with a volume submap algorithm. The branch is in the shape of a figure eight to show how the grafting algorithm handles the cusps. The results of the algorithm can be seen in Figure 10. The grafting algorithm again produced high quality elements at the bounding loop of the graft surface. This time there were no quality issues outside of the graft surface.

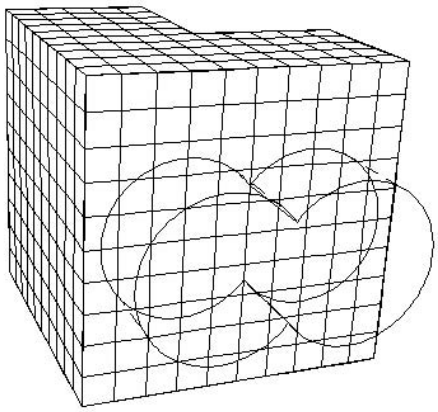

Figure 9. Submapped trunk with figure eight branch before grafting.

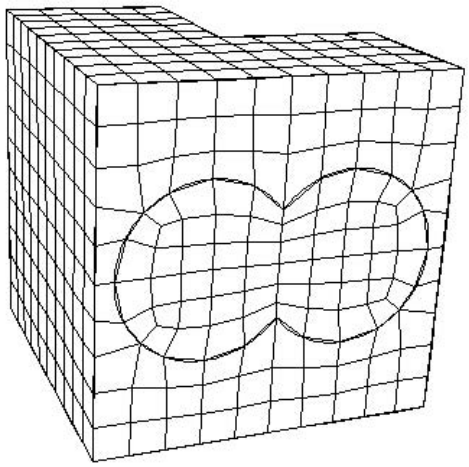

Figure 10. Cut away view of the base surface after grafting.

Finally, Figure 11 shows a trunk with a swept volume mesh. Notice that there is a through hole down the center of the trunk. The heart-shaped branch protruding from the trunk complicates the meshing of the solid. It cannot be easily meshed with any of the structured meshing algorithms. The grafting algorithm is able to produce a high quality mesh on the solid. Figure 12 shows the base surface after grafting the mesh.

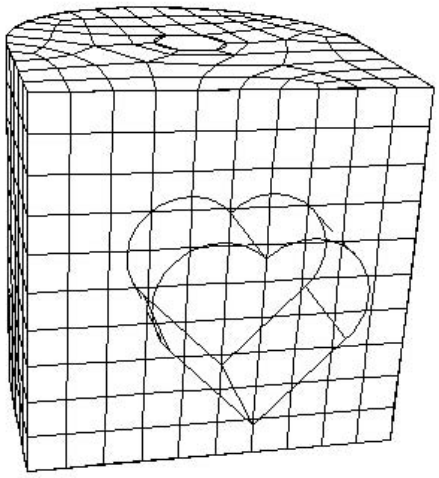

Figure 11. Swept volume with through hole and heart-shaped branch before grafting.

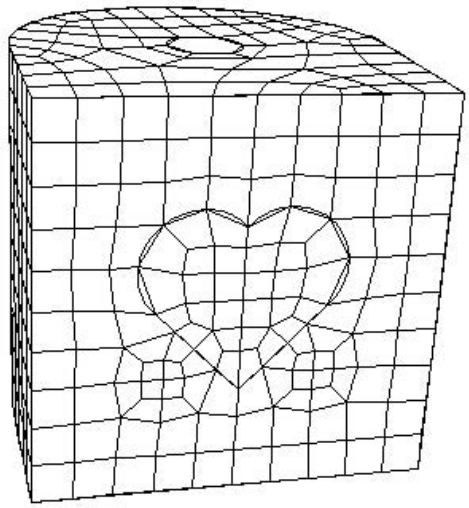

Figure 12. Cut away view of base surface after grafting.

5. CONCLUSION

Sweeping algorithms have become very mature over the last several years. However, these algorithms require that the linking surfaces be mappable or submappable relying on the unstructured techniques to mesh the rest. With the grafting algorithm, an additional set of solids can be meshed in a structured way. The grafting algorithm modifies the linking surfaces of a swept trunk to create a high quality transition to the sweepable branches.

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