Arctic Sea Ice Modeling with the Material-Point Method

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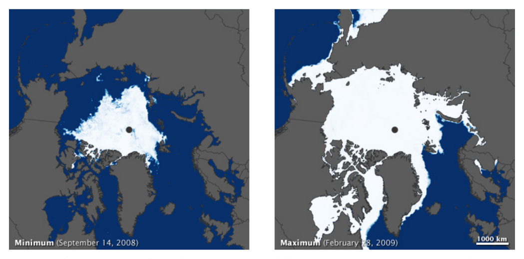
Introduction

- Arctic sea ice plays an important role in global climate by reflecting solar radiation and insulating the ocean from the atmosphere
- Due to feedback effects, the Arctic sea ice cover is changing rapidly
- To accurately model this change, high-resolution calculations must incorporate
  - annual cycle of growth and melt due to radiative forcing
  - mechanical deformation due to surface winds, ocean currents and Coriolis forces
  - localized effects of leads and ridges.



Composite satellite images of ice compactness for 2008 minimum (left) and 2009 maximum (right). Source: earthobservatory.nasa.gov.

Thickness variations in the ice. Source: alaska.usgs.gov.

Sea ice pressure ridge. Source: arctic.noaa.gov.

Sea Ice Governing Equations

- 2-D momentum equation for ice velocity ($\mathbf{v}$):

$$\rho \bar{h}\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = \nabla\cdot(\bar{h}\sigma) + \mathbf{t}_a + \mathbf{t}_w - \mathbf{f}_c$$

Coriolis Force: $\mathbf{f}_c = 2\rho\bar{h}\omega \sin\phi(e_3 \times \mathbf{v})$
Atmospheric Drag: $\mathbf{t}_a = C_a\|\mathbf{v}_a\|\mathbf{v}_a$
Ocean Drag: $\mathbf{t}_w = C_w\|\mathbf{v}-\mathbf{v}_w\|(\mathbf{v}-\mathbf{v}_w)$
Stress: σ

- Ice thickness distribution (g) for variations in thickness (h):

$$\frac{\partial g}{\partial t} + \nabla\cdot(\mathbf{v}g) + \frac{\partial (fg)}{\partial h} = \psi$$

Average thickness: $\bar{h} = \int_0^\infty hgdh$

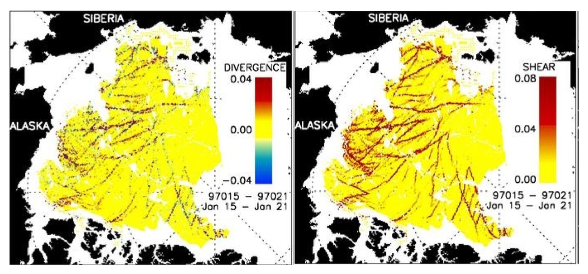
- 1-D vertical heat equation for temperature (T) and change thickness due to growth and melt ($f = \partial h/\partial t$):

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \kappa I_0 e^{-\kappa z}$$

Flux balance at ocean: $F_w - k\frac{\partial T}{\partial z} = -q\frac{\partial h}{\partial t}$
Flux balance at atmosphere: $F_a + k\frac{\partial T}{\partial z} = -q\frac{\partial h}{\partial t}$

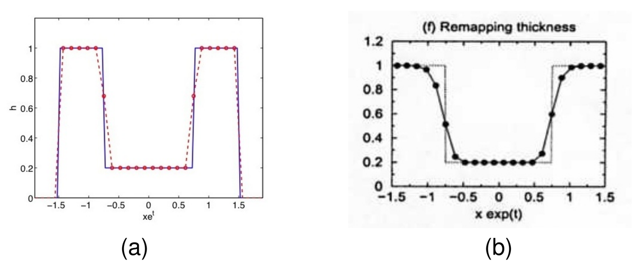
Motivation for New Model

- Isotropic rheology is not appropriate for high-resolutions where a single lead can dominate the deformation



Radarsat Geophysical Processor System (RGPS) velocity divergence and shear displaying linear kinematic features associated with leads. Source: rkwok.jpl.nasa.gov

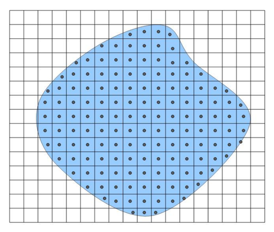
- Artificial diffusion in transport algorithms can lead to errors in thickness and smearing of ice edge



Thickness transport solutions in a convergent velocity field for (a) MPM (b) remapping algorithm [Lipscomb and Hunke 2004]

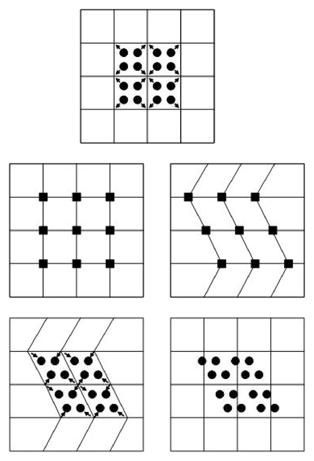
Material-Point Method (MPM) [Sulsky *et al.* 2007]

- Domain divided into material points and background grid
- Lagrangian material points carry mass, momentum, thickness distribution, and internal variables for constitutive model
- Momentum equation solved on background grid using FEM



Domain for MPM calculation.

- Algorithm
  - Map MP values to nodes
  - Calculate internal and external forces at nodes
  - Solve momentum equation on grid
  - Update MPs based on grid solution
  - Evaluate constitutive model at MPs
  - Update ITD based on mechanics/thermodynamics
  - Regrid

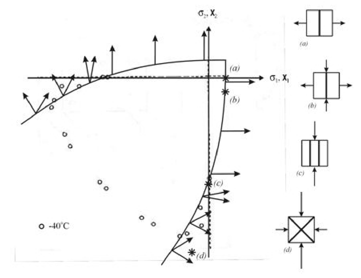


MPM algorithm steps.

Elastic-Decohesive Constitutive Model [Schreyer *et al.* 2006]

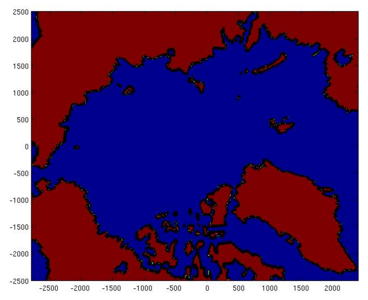
- Leads modeled as displacement discontinuities
- Intact ice modeled as elastic
- Predicts initiation and orientation of lead

Leads in the ice.

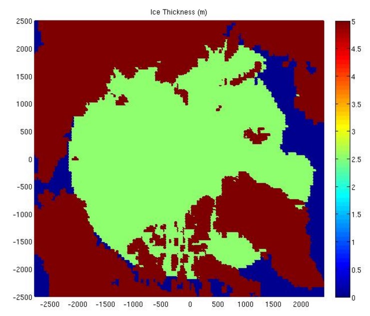


Elastic-decohesive failure curve in principal stress space. Arrows indicate normal direction of discontinuity. Data points from Schulson [Schulson 2001]

Pan-Arctic Calculation



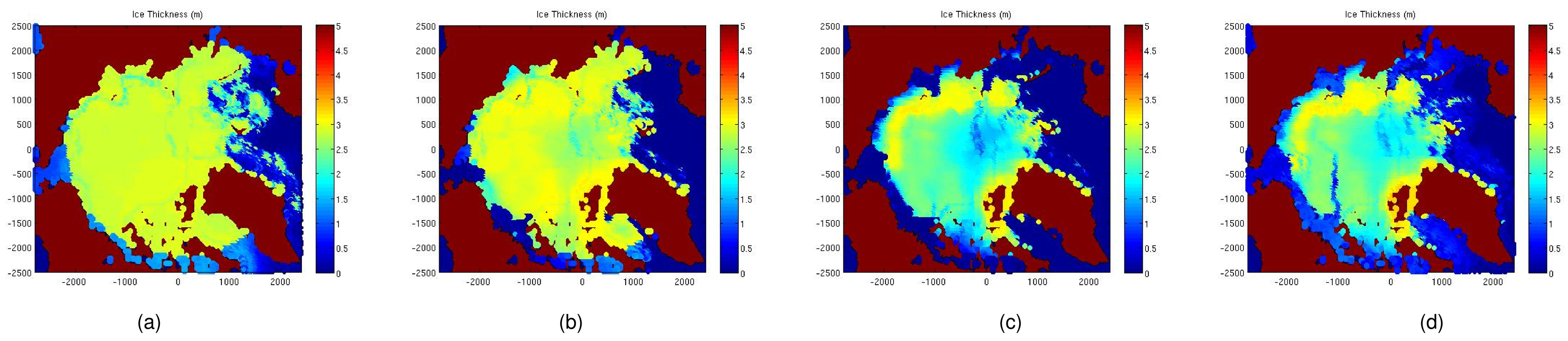
Calculation domain with land colored red and ocean blue.



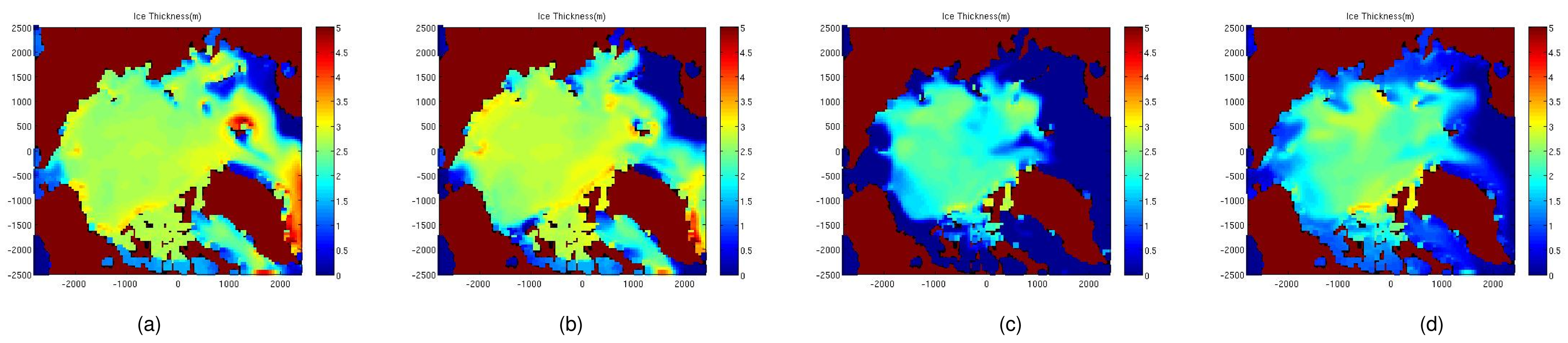
Ice thickness initial configuration.

- 50 km resolution rectangular grid (azimuthal equal area projection)
- Initialization - 2.5 m ice above 70 degrees N latitude
- MPM seaice model run with
  - Elastic-decohesive rheology
  - Energy-conserving thermodynamics
  - Five category ice thickness distribution
- Compare with LANL CICE code [Hunke and Lipscomb 2006]

First Year Comparison

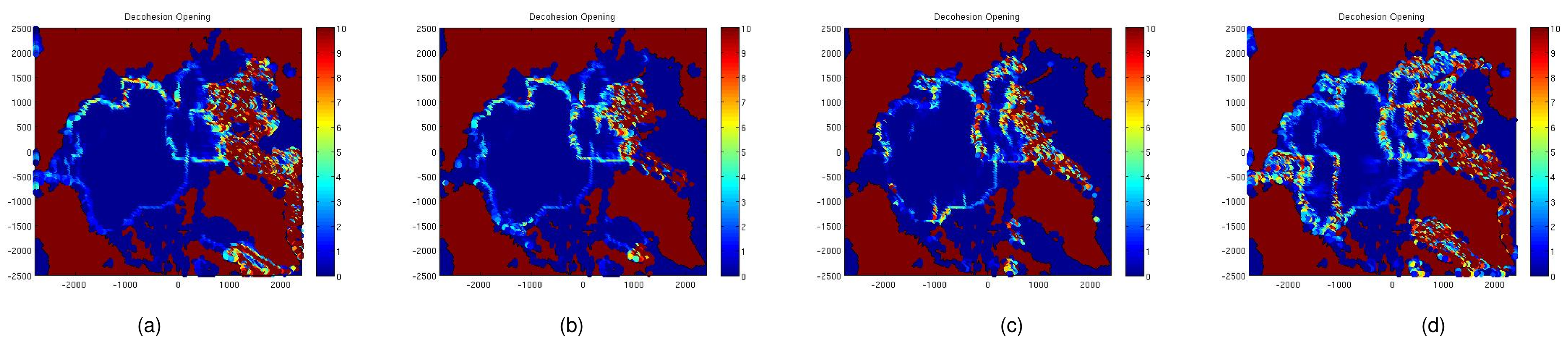


MPM results for ice thickness on (a) March, (b) June, (c) September and (d) December



LANL CICE results for ice thickness on (a) March, (b) June, (c) September and (d) December

Discontinuties



MPM results for normalized opening magnitude on (a) March, (b) June, (c) September and (d) December

Conclusions

We have demonstrated a new mathematical algorithm for solving the sea ice governing equations using the material-point method with an elastic-decohesive constitutive model. An initial comparison with the LANL CICE code indicates that the ice edge is sharper using MPM, but that many of the overall features are similar.

References

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