Optimizing coherent quantum feedback network for squeezed-light generation

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Motivation

- Squeezed light can help to increase the secure communication distance and/or key rate for continuous-variable quantum key distribution (CV-QKD). High-rate CV-QKD requires high-bandwidth squeezing spectrum (i.e., strong squeezing at bandwidths of ~ 100 MHz or even ~ 1 GHz around the carrier).
- Squeezed light is also a fundamental pre-requisite for generation of entanglement in CV quantum repeaters and CV cluster-state (measurement-based) quantum computation.

Scope of the work

- We consider a network of coupled linear and bilinear optical elements such as mirrors, beam-splitters, phase-shifters, lasers, and optical parametric oscillators (OPOs).
- The idea is to use such a coherent quantum feedback network (CQFN) to generate the output light field with a favorable squeezing spectrum.
- The objective is to maximize the degree of squeezing at a chosen bandwidth frequency (or a range of frequencies) by searching over the space of model parameters with experimentally motivated bounds.

The $(\mathbf{S}, \mathbf{L}, H)$ model of an optical CQFN

- Let n be the number of the network's input/output ports and m be the number of cavities (assuming one internal field mode per cavity).
- Let $\mathbf{a}$, $\mathbf{a}_{\text{in}}$, and $\mathbf{a}_{\text{out}}$ denote, respectively, vectors of boson annihilation operators for the cavity modes, input fields, and output fields: $\mathbf{a} = [a_1, \ldots, a_m]^{\mathsf{T}}$, $\mathbf{a}_{\text{in}} = [a_{\text{in},1}, \ldots, a_{\text{in},n}]^{\mathsf{T}}$, $\mathbf{a}_{\text{out}} = [a_{\text{out},1}, \ldots, a_{\text{out},n}]^{\mathsf{T}}$.
- The CQFN is fully described by the $(\mathbf{S}, \mathbf{L}, H)$ model, which includes:
  $\mathbf{S}$ is an $n \times n$ matrix that describes the scattering of external fields;
  $\mathbf{L}$ is an $n \times 1$ matrix that describes the coupling of cavity modes and external fields;
  H is the Hamiltonian that describes the intracavity dynamics.
- The quantum Langevin equations for the cavity mode operators $\{a_\ell(t)\}$ are ($\hbar = 1$)
$$\frac{da_\ell}{dt} = -i[a_\ell, H] + \mathcal{L}_L[a_\ell] + \Gamma_l, \quad \ell = 1, \ldots, m, \tag{1}$$
where $\mathcal{L}_L$ is the Lindblad superoperator and Γ_l is the noise operator.
- The generalized boundary condition for the network is $\mathbf{a}_{\text{out}} = \mathbf{S}\mathbf{a}_{\text{in}} + \mathbf{L}$.
- The elements of $\mathbf{L}$ are linear in annihilation operators of the cavity modes: $\mathbf{L} = \mathbf{K}\mathbf{a}$, and the Hamiltonian has the bilinear form:
$$H = \mathbf{a}^{\dagger}\boldsymbol{\Omega}\mathbf{a} + \tfrac{i}{2}\mathbf{a}^{\dagger}\mathbf{W}\mathbf{a}^{\ddagger} - \tfrac{i}{2}\mathbf{a}^{\mathsf{T}}\mathbf{W}^{\dagger}\mathbf{a}, \tag{2}$$
where $\mathbf{a}^{\dagger} = [a_1^{\dagger}, \ldots, a_m^{\dagger}]$ and $\mathbf{a}^{\ddagger} = \mathbf{a}^{\dagger\mathsf{T}}$.
- With such $\mathbf{L}$ and H, the quantum Langevin equations (1) take the matrix form:
$$\frac{d\mathbf{a}}{dt} = \mathbf{V}\mathbf{a} + \mathbf{W}\mathbf{a}^{\ddagger} + \mathbf{Y}\mathbf{a}_{\text{in}}, \tag{3}$$
where $\mathbf{V} = -\frac{1}{2}\mathbf{K}^{\dagger}\mathbf{K} - i\boldsymbol{\Omega}, \quad \mathbf{Y} = -\mathbf{K}^{\dagger}\mathbf{S}$.

The model of CQFN in the frequency domain

- Boson operators in the frequency domain:
$$b(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega\, b(\omega)e^{-i\omega t}, \quad b^{\dagger}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega\, b^{\dagger}(-\omega)e^{-i\omega t},$$
where $b(t)$ stands for any element of $\mathbf{a}(t)$, $\mathbf{a}_{\text{in}}(t)$, and $\mathbf{a}_{\text{out}}(t)$.
- The double-length column vectors notation: $\breve{\mathbf{b}}(\omega) = \begin{bmatrix} \mathbf{b}(\omega) \\ \mathbf{b}^{\ddagger}(-\omega) \end{bmatrix}$, where $\mathbf{b}(\omega)$ stands for either of $\mathbf{a}(\omega)$, $\mathbf{a}_{\text{in}}(\omega)$, and $\mathbf{a}_{\text{out}}(\omega)$.
- The quantum input-output relations in the matrix form:
$$\breve{\mathbf{a}}_{\text{out}}(\omega) = \breve{\mathbf{Z}}(\omega)\breve{\mathbf{a}}_{\text{in}}(\omega), \tag{4}$$
where $\breve{\mathbf{Z}}(\omega)$ is the network's transfer-matrix function:
$$\breve{\mathbf{Z}}(\omega) = \begin{bmatrix} \mathbf{Z}^{-}(\omega) & \mathbf{Z}^{+}(\omega) \\ \mathbf{Z}^{+}(-\omega)^{*} & \mathbf{Z}^{-}(-\omega)^{*} \end{bmatrix} = \left[\mathbf{I}_{2n} + \breve{\mathbf{K}}(\breve{\mathbf{A}} + i\omega\mathbf{I}_{2m})^{-1}\breve{\mathbf{K}}^{\dagger}\right]\breve{\mathbf{S}}. \tag{5}$$
Here, $\breve{\mathbf{A}} = \Delta(\mathbf{V}, \mathbf{W})$, $\breve{\mathbf{K}} = \Delta(\mathbf{K}, \mathbf{0})$, $\breve{\mathbf{S}} = \Delta(\mathbf{S}, \mathbf{0})$, and we use the notation: $\Delta(\mathbf{A}, \mathbf{B}) = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{*} & \mathbf{A}^{*} \end{bmatrix}$.

The squeezing spectrum

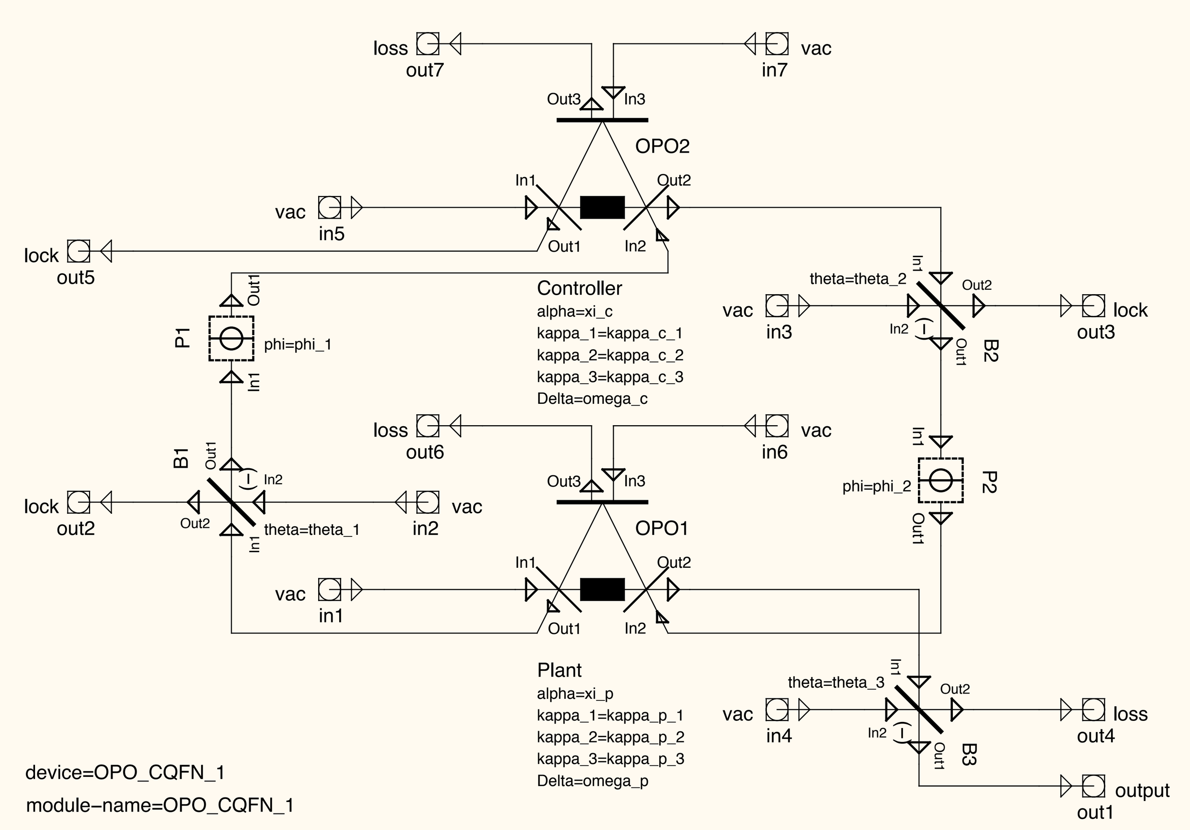
- The power spectrum density of the quadrature's quantum noise (*squeezing spectrum*):
$$\mathcal{P}_i(\omega, \theta) = 1 + \int_{-\infty}^{\infty} d\omega' \langle : X_i(\omega, \theta), X_i(\omega', \theta) : \rangle, \tag{6}$$
where $X_i(\omega, \theta) = a_{\text{out},i}(\omega)e^{-i\theta} + a_{\text{out},i}^{\dagger}(-\omega)e^{i\theta}$ is the quadrature of the ith output field in the frequency domain, θ is the homodyne phase, $::$ is the normal ordering of boson operators, and $\langle x, y \rangle = \langle xy \rangle - \langle x \rangle\langle y \rangle$.
- Using the $(\mathbf{S}, \mathbf{L}, H)$ model of the CQFN, we obtain:
$$\mathcal{P}_i(\omega, \theta) = 1 + \mathcal{N}_i(\omega) + \mathcal{N}_i(-\omega) + \mathcal{M}_i(\omega)e^{-2i\theta} + \mathcal{M}_i(\omega)^{*}e^{2i\theta}, \tag{7}$$
$$\mathcal{N}_i(\omega) = \int_{-\infty}^{\infty} d\omega' \langle a_{\text{out},i}^{\dagger}(-\omega')a_{\text{out},i}(\omega) \rangle = \sum_{j=1}^{n} \left|Z_{ij}^{+}(\omega)\right|^2, \tag{8}$$
$$\mathcal{M}_i(\omega) = \int_{-\infty}^{\infty} d\omega' \langle a_{\text{out},i}(\omega)a_{\text{out},i}(\omega') \rangle = \sum_{j=1}^{n} Z_{ij}^{-}(\omega)Z_{ij}^{+}(-\omega). \tag{9}$$
- We are only interested in the squeezing spectrum of the field at one of the output ports (designated as $i = 1$): $\mathcal{P}(\omega, \theta) = \mathcal{P}_1(\omega, \theta)$.

The measure of squeezing

- The squeezing figure of merit measured in decibels is
$$\mathcal{Q}(\omega, \theta) = 10\log_{10}\mathcal{P}(\omega, \theta). \tag{10}$$
- The maximum and minimum of $\mathcal{P}(\omega, \theta)$ as a function of θ,
$$\mathcal{P}^{+}(\omega) = \max_{\theta}\mathcal{P}(\omega, \theta), \quad \mathcal{P}^{-}(\omega) = \min_{\theta}\mathcal{P}(\omega, \theta), \tag{11}$$
are spectra of the quantum noise in anti-squeezed and squeezed quadrature, respectively. The corresponding logarithmic spectral measures of anti-squeezing and squeezing:
$$\mathcal{Q}^{\pm}(\omega) = 10\log_{10}\mathcal{P}^{\pm}(\omega). \tag{12}$$
- Using Eq. (7), we find:
$$\mathcal{P}^{\pm}(\omega) = 1 + \mathcal{N}(\omega) + \mathcal{N}(-\omega) \pm 2|\mathcal{M}(\omega)|. \tag{13}$$

The model of a network of two coupled OPOs

- A schematic depiction of the CQFN of two coupled OPOs:



- Parameters of the CQFN of two coupled OPOs:

Parameter	Type	Description
κ_{p1}	Positive	Leakage rate for left mirror of plant cavity
κ_{p2}	Positive	Leakage rate for right mirror of plant cavity
κ_{p3}	Positive	Leakage rate for losses in plant cavity
ω_{p}	Real	Frequency detuning of plant cavity
ξ_{p}	Complex	Pump amplitude of plant OPO
κ_{c1}	Positive	Leakage rate for left mirror of controller cavity
κ_{c2}	Positive	Leakage rate for right mirror of controller cavity
κ_{c3}	Positive	Leakage rate for losses in controller cavity
ω_{c}	Real	Frequency detuning of controller cavity
ξ_{c}	Complex	Pump amplitude of controller OPO
ϕ_1	Real	Phase shift of the first phase shifter
ϕ_2	Real	Phase shift of the second phase shifter
θ_1	Real	Rotation angle of the first beamsplitter
θ_2	Real	Rotation angle of the second beamsplitter
θ_3	Real	Rotation angle of the third beamsplitter

Physical description of the CQFN of two coupled OPOs

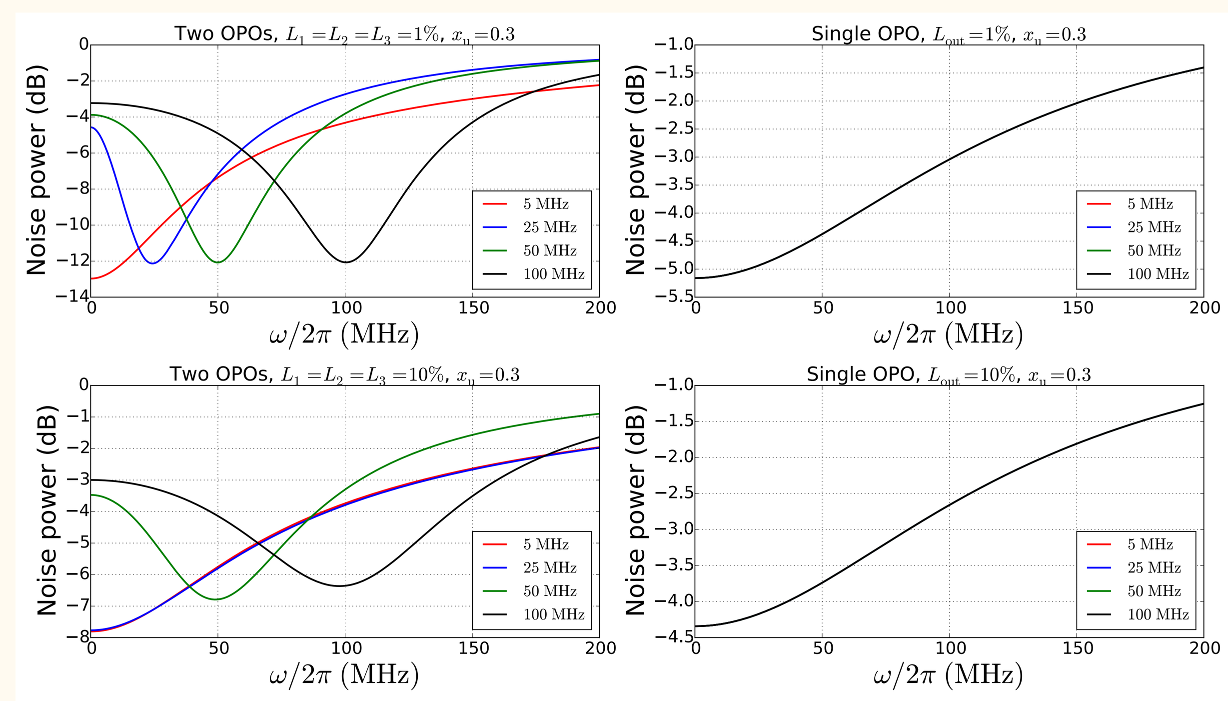
- Pump fields for both OPOs are assumed to be classical and not shown in the scheme.
- From the control theory perspective, OPO1 is considered to be the *plant* and OPO2 the (quantum) *controller*.
- Each OPO cavity has a fictitious third mirror to model intracavity losses.
- Beamsplitters B1 and B2 represent the light diverted to lock the cavities as well as losses in optical transmission lines between the OPOs. Beamsplitter B3 represents losses in the output transmission line and detection inefficiencies.
- Phase shifters P1 and P2 are inserted into transmission lines between the OPOs to manipulate the interference underlying the CQF control.
- Taking into account the feedback loop between the plant and controller, the CQFN has seven input ports, seven output ports, and two cavity modes ($n = 7$, $m = 2$).
- With $\xi_{\text{p}} = |\xi_{\text{p}}|e^{i\theta_{\text{p}}}$ and $\xi_{\text{c}} = |\xi_{\text{c}}|e^{i\theta_{\text{c}}}$, the CQFN is described by 17 real parameters (five of which correspond to losses).
- The relationship between leakage rate and power transmittance of a mirror:
$$\kappa_i = cT_i/(2l_{\text{eff}}), \quad i = 1, 2, 3, \tag{14}$$
where T_i is the power transmittance of the ith mirror ($R_i = 1 - T_i$ is the power reflectance), c is the speed of light, and l_{eff} is the effective cavity length.
- The total leakage rate (including losses) from the plant and controller cavities: $\gamma_{\text{p}} = \kappa_{\text{p1}} + \kappa_{\text{p2}} + \kappa_{\text{p2}}$ and $\gamma_{\text{c}} = \kappa_{\text{c1}} + \kappa_{\text{c2}} + \kappa_{\text{c2}}$.
- The scaled pump amplitude for the plant and controller OPOs:
$$x_{\text{p}} = 2|\xi_{\text{p}}|/\gamma_{\text{p}} = \sqrt{P_{\text{p}}/P_{\text{p,th}}}, \quad x_{\text{c}} = 2|\xi_{\text{c}}|/\gamma_{\text{c}} = \sqrt{P_{\text{c}}/P_{\text{c,th}}}, \tag{15}$$
where P is the OPO pump power and P_{th} is its threshold value.
- The QNET package (developed by Hideo Mabuchi's group at Stanford University) is used to derive the $(\mathbf{S}, \mathbf{L}, H)$ model of the CQFN.

Numerical optimization results for the squeezing spectrum

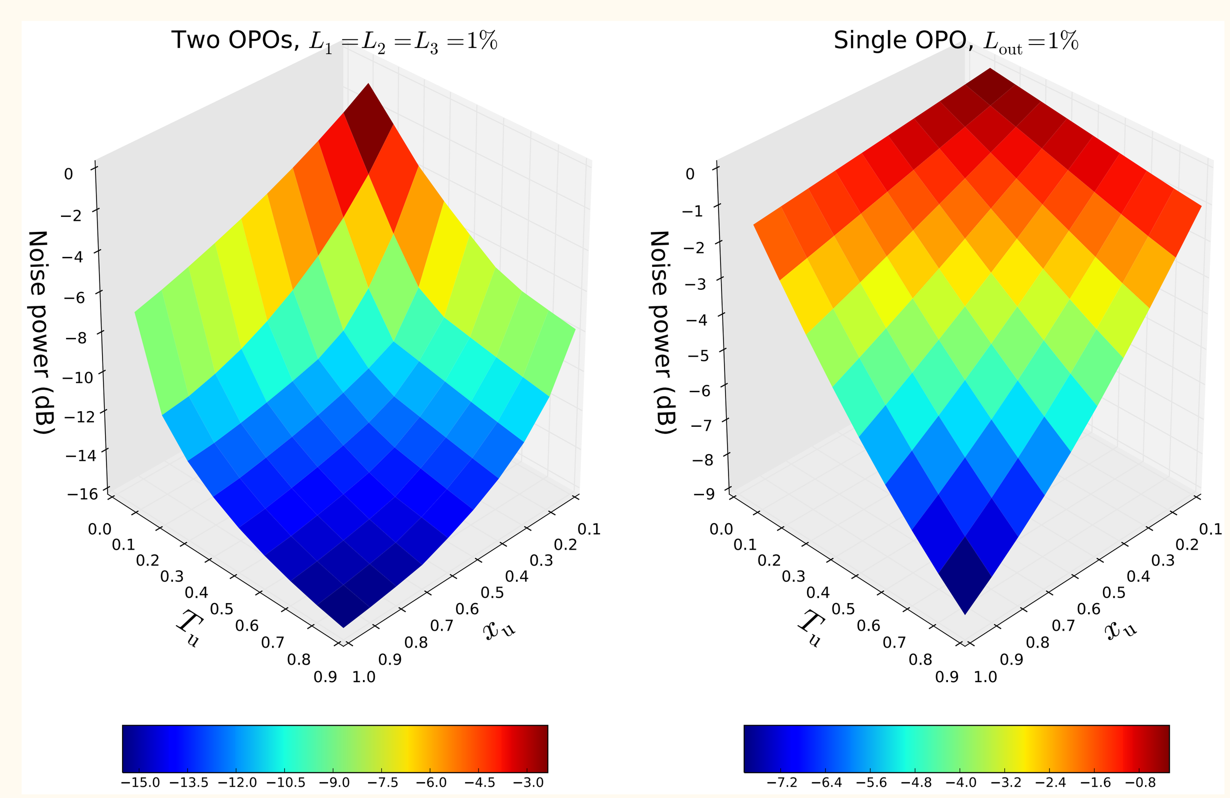
- We maximize the degree of squeezing (i.e., minimize $\mathcal{P}^{-}(\omega)$) at a chosen bandwidth frequency $\omega_{\text{opt}}/2\pi = \{5, 25, 50, 100\}$ MHz.
- We use a suite of global (stochastic) algorithms (PyGMO, http://esa.github.io/pygmo). We run in parallel 8 global algorithms to maximize the chance of finding a globally optimal parameter set.
- Optimizations are performed for various values of intracavity losses $L_{\text{in}} = T_{\text{p3}} = T_{\text{c3}}$, transmission losses $L_i = \sin^2(\theta_i)$ $(i = 1, 2, 3)$, upper bound T_{u} on power transmittances of cavity mirrors, and upper bound x_{u} on scaled pump amplitudes of OPOs.
- Maximized degree of squeezing (in dB) found using various algorithms (for $L_{\text{in}} = 0.01$, $L_1 = L_2 = L_3 = 0.01$, $T_{\text{u}} = 0.2$, $x_{\text{u}} = 0.8$):

Algorithm	Frequency [MHz]			
	5	25	50	100
Sequential Least Squares Programming (local only)	-10.9275	-10.7843	-7.5043	-2.6008
Monotonic Basin Hopping with Sequential Least Squares Programming	-12.5509	-10.8660	-9.8984	-3.6246
Monotonic Basin Hopping with Compass Search	-9.8994	-6.8967	-5.9558	-5.1477
Covariance Matrix Adaptation Evolution Strategy	-11.9805	-10.7991	-9.3718	-3.6028
Particle Swarm Optimization	-11.8080	-9.3243	-6.2234	-3.6183
Improved Harmony Search	-11.3148	-9.3594	-8.4055	-3.6047
Differential Evolution	-12.5508	-10.8662	-9.8984	-3.6246
Differential Evolution with p-best crossover	-12.5504	-10.8664	-9.8984	-3.6246
Artificial Bee Colony	-12.5477	-10.8625	-9.8791	-8.9308
Set of 8 global algorithms run in parallel	-12.5513	-10.8662	-9.8984	-8.9315

- Optimized squeezing spectrum for the CQFN of two coupled OPOs compared to that for a single OPO (for loss and bound values $L_{\text{in}} = 0.01$, $L_1 = L_2 = L_3 = \{0.01, 0.1\}$, $T_{\text{u}} = 0.9$, $x_{\text{u}} = 0.3$):

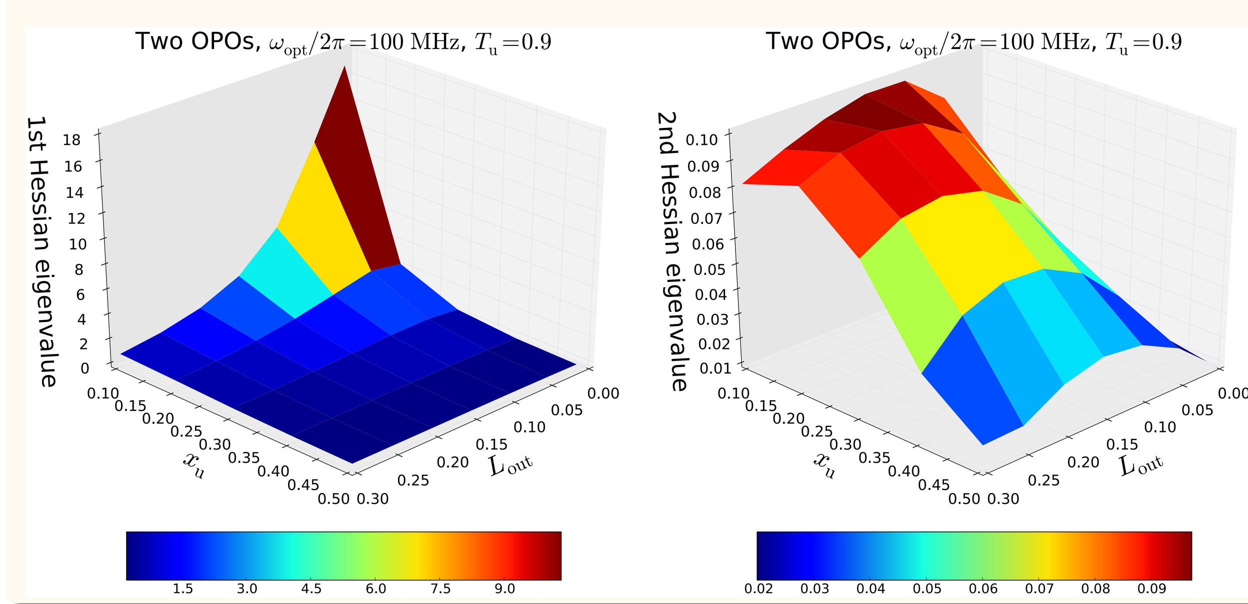


- Maximized degree of squeezing at $\omega_{\text{opt}}/2\pi = 100$ MHz as a function of upper bounds T_{u} and x_{u} (for $L_{\text{in}} = 0.01$ and $L_1 = L_2 = L_3 = 0.01$):



Robustness of optimal CQFN configurations

- We investigate the robustness of optimal CQFN configurations to small variations in phase parameters $\{\theta_{\text{p}}, \theta_{\text{c}}, \phi_1, \phi_2\}$ by analyzing eigenvalues of the Hessian matrix. Two of the four eigenvalues are always zero up to numerical precision. Non-zero Hessian eigenvalues versus x_{u} and $L_{\text{out}} = L_1 = L_2 = L_3$ (for $\omega_{\text{opt}}/2\pi = 100$ MHz, $L_{\text{in}} = 0.01$, $T_{\text{u}} = 0.9$):



Conclusions

- The $(\mathbf{S}, \mathbf{L}, H)$ model makes it possible to evaluate the squeezing spectrum for various values of experimental parameters.
- Use of global search methods is critical for finding the best possible performance of the CQFN, especially for squeezing at higher bandwidths.
- The CQFN of two coupled OPOs makes it possible to vary the squeezing spectrum, effectively utilize available pump power, and overall significantly outperform a single OPO.
- The squeezing generation performance of optimal configurations of the CQFN is robust to small variations of phase parameters.

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