Calibrating two-qubit gates via robust phase estimation

Kenneth Rudinger¹, Guilhem Ribeill², Luke Govia², Matthew Ware², Shelby Kimmel³

¹Center for Computing Research, Sandia National Laboratories
²BBN Technologies
³Middlebury College

APS March Meeting 2019
Boston, MA
Main results

- We used robust phase estimation (RPE) to estimate the phase of a two-qubit cross-resonance gate on superconducting transmon qubits.
- Decoherence makes “naïve” RPE results untrustworthy.
- Can leverage RPE’s “robustness” promise to determine when phase estimates are actually to be trusted.
- *If your two-qubit gates don’t induce too much depolarization, you can use two-qubit RPE to calibrate your two-qubit gate rotation angle today.*
Motivation

We want full scale universal quantum computation!

- Need:
  - Quantum error correction.
  - Fault-tolerant operations.
  - Coherent control.
  - Calibration techniques for coherent operations that are robust, efficient, and accurate.

- How do we calibrate coherent operations?
  - Ramsey/Rabi experiments!
  - RPE is “Ramsey/Rabi++”
RPE as Rabi/Ramsey++

- Kimmel, Low, Yoder: Phys. Rev. A 92, 062315
- RPE- Estimate gate phase by combining Rabi and Ramsey experiments.
- Improve estimate with logarithmically spaced sequences.
- Provably robust against SPAM errors, gate errors.
- Efficient- few samples needed per sequence.
- Accurate- $L^{-1}$ accuracy scaling (Heisenberg scaling).
  - Demonstrated experimentally on 1-qubit gates in trapped ions
  - Accuracy of $10^{-4}$ radians with 176 experiments total.
How RPE works

- Suppose I have noisy gate $U$ implementing $X_\phi = e^{-i\phi X/2} \ (\phi \approx \pi/2)$, along with noisy $|0\rangle$ prep and measurement:

\[
|\langle 0 | U^L | 0 \rangle|^2 = \frac{1}{2} (1 + \cos L\phi) + \delta_c(L)
\]

\[
|\langle 0 | U^{L+1} | 0 \rangle|^2 = \frac{1}{2} (1 - \sin L\phi) + \delta_s(L)
\]

- If $\delta < 1/\sqrt{8} \approx 0.35$, can estimate $\phi$ to within $\pi/(2L)$!
- WLOG, can generalize to arbitrary gates on arbitrary number of qubits.
How well can we estimate $\phi$?

- Start with $L=1$ and increment. At each $L$, can rule out half of remaining angular space.

- Intuition: Amplify coherent errors by repeating gates.
- Provably optimal!
- Additive noise ($\delta$) will bias estimates up to $\pi/(2L)$.
- Works great for one qubit; let’s do this for two qubits!
Experimental device

5-transmon device
Courtesy IBM

<table>
<thead>
<tr>
<th>Qubit</th>
<th>$T_1$ (μs)</th>
<th>$T_2$ (μs)</th>
<th>$T_2^*$ (μs)</th>
<th>$f_{10}$ (GHz)</th>
<th>$T_{gate}$ (ns)</th>
<th>$r_{gate}$</th>
<th>$F_{RO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>66</td>
<td>26</td>
<td>43</td>
<td>5.053</td>
<td>60</td>
<td>1.2e-3</td>
<td>81.6</td>
</tr>
<tr>
<td>Q3</td>
<td>26</td>
<td>15</td>
<td>36</td>
<td>5.202</td>
<td>60</td>
<td>6.8e-3</td>
<td>83.9</td>
</tr>
</tbody>
</table>

1 Single-qubit gate  
2 Single-qubit error per Clifford from Clifford RB  
3 Single-qubit readout fidelity  
4 Python DSL for quantum gates:  
   https://github.com/BBN-Q/QGL  
Experimental device

Cross-resonance gate:

\[ U = e^{-\frac{i\varphi}{2} Z \otimes X} | \varphi = \frac{\pi}{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 & 0 \\ -i & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & i & 1 \end{pmatrix} \]

\varphi \text{ is tuned by adjusting pulse duration or amplitude.}
Let’s try it!

- Can we characterize $\varphi$ for varying pulse amplitudes?

$$|\langle 1 + y |U^L|10 \rangle|^2 = \frac{1 - \sin L\varphi}{2}$$
$$|\langle 1 + y |U^L|10 \rangle|^2 = \frac{1 + \cos L\varphi}{2}$$

- $L=1,2,4,...,64$
- 14 unique circuits!
Let’s try it!

- Can we characterize $\varphi$ for varying pulse amplitudes?

\[
|\langle 1 + y | U^L | 10 \rangle|^2 = \frac{1 - \sin L\varphi}{2} \quad |\langle 1 + y | U^L | 10 \rangle|^2 = \frac{1 + \cos L\varphi}{2}
\]

- $L=1,2,4,...,64$
- 14 unique circuits!
Let’s try it!

- Can we characterize $\varphi$ for varying pulse amplitudes?

$$|\langle 1 + y | \mathbf{U}^L | 10 \rangle|^2 = \frac{1 - \sin L \varphi}{2} \quad |\langle 1 + y | \mathbf{U}^L | 10 \rangle|^2 = \frac{1 + \cos L \varphi}{2}$$

- $L=1,2,4,...,64$
- 14 unique circuits!
Let’s try it!

- Can we characterize $\varphi$ for varying pulse amplitudes?

$$|\langle 1 + y | U^L | 10 \rangle|^2 = \frac{1 - \sin L\varphi}{2} \quad \text{and} \quad |\langle 1 + y | U^L | 10 \rangle|^2 = \frac{1 + \cos L\varphi}{2}$$

- $L = 1, 2, 4, \ldots, 64$
- 14 unique circuits!
Let’s try it!

- Can we characterize $\varphi$ for varying pulse amplitudes?

$$\left| \langle 1 + y | U^L | 10 \rangle \right|^2 = \frac{1 - \sin \varphi}{2} \quad \left| \langle 1 + y | U^L | 10 \rangle \right|^2 = \frac{1 + \cos \varphi}{2}$$

- $L=1,2,4,...,64$
- 14 unique circuits!
Let’s try it!

- Can we characterize $\phi$ for varying pulse amplitudes?
  \[
  \left| \langle 1 + y | U^L | 10 \rangle \right|^2 = \frac{1 - \sin L\phi}{2} \quad \left| \langle 1 + y | U^L | 10 \rangle \right|^2 = \frac{1 + \cos L\phi}{2}
  \]

- $L=1,2,4,...,64$
- 14 unique circuits!

...uh-oh. What happened?!
What happened?!

- CR gate is 0.8 μs. T1≈26 μs. 64 CR gates... totally depolarized!
- RPE failed because δ > 35%.
- We can try to use external calibration info to tell us when to stop increasing k, but this seems... suboptimal.
- Instead: We can estimate δ. Stop trusting RPE once δ > 35%.
- At each L’, have estimate φ(L’). For L ≤ L’ ask:
  \[
  \left| \frac{1 + \cos L \varphi(L')}{2} - f_L^{\cos} \right| \leq 35\%?
  \]
  \[
  \left| \frac{1 - \sin L \varphi(L')}{2} - f_L^{\sin} \right| \leq 35\%?
  \]
$\delta > 35\%$?

$\phi_{L'} = 1.49; L' = 1$

- observed
- $(1 - \sin(L\phi_{L'}))/2$
\[ \delta > 35\% ? \]

\[ \phi_{L'} = 1.08; L' = 2 \]

\[ \text{observed} \]

\[ \frac{1 - \sin(L\phi_{L'})}{2} \]

Pr. of observing 00

CR repetitions (L)
\[ \delta > 35\%? \]

\[ \phi_{L'} = 0.77; L' = 4 \]

- observed
- \((1 - \sin(L\phi_{L'}))/2\)

Pr. of observing 00 vs. CR repetitions (L)
$\delta > 35\%$?

$\phi_{L'} = 1.05; L' = 8$

- Green diamond: observed
- Orange diamond: $(1 - \sin(L\phi_{L'}))/2$

~45%
Now we can trust RPE estimates. Here only good maximally for 4 CR repetitions (maximum error bars $\pi/8$).
Conclusions

- RPE has self-consistency check built in; easy to evaluate.
- You can use RPE today to calibrate your two-qubit gates, but beware of decoherence errors!
- Can use pyGSTi to do the analysis, but sequences not yet built in for arbitrary two-qubit gates. Easy to figure out - email me!
  - [www.pygsti.info](http://www.pygsti.info)
  - kmrudin@sandia.gov
- Thank you!

Obligatory Icon Needs Quantum acronym (OINQY)
Bonus slides
Experimental Details: Two-Qubit Gate

Standard echoed cross-resonance gate

- Implements ZX(-90) rotation
- Drive control qubit (Q1) at target qubit’s (Q3) frequency
- Echo to cancel out IX, ZI cross-talk terms

Varied CR pulse length and amplitude to change angle of ZX90

2-qubit Randomized Benchmarking

Error per Clifford: 0.072

\[
\frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -i & 0 & 0 \\
-i & 1 & 0 & 0 \\
0 & 0 & 1 & i \\
0 & 0 & i & 1
\end{pmatrix}
\]

\[|11\rangle \quad \overset{(+)}{\text{control}} \quad \overset{(+)}{\text{target}} \quad \overset{(-)}{\tilde{\omega}_1} \quad \overset{(-)}{\tilde{\omega}_2} \quad |10\rangle
\]

\[|01\rangle \quad \overset{(+)}{\tilde{\omega}_1} \quad \overset{(+)}{\text{target}} \quad \overset{(-)}{\tilde{\omega}_2} \quad |00\rangle
\]

Sweeping CR time instead of amplitude

![Graph showing the relationship between angle estimate and half CR pulse duration (ns). The graph includes data points for different values of k (1, 2, 4, 8, 16, 32, 64) and a horizontal line indicating π/2.](image-url)