On the Calculation of Radar Cross Sections for Multilayered Spherical Targets

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Outline of Presentation

A look at the RCS of a Metal Sphere (definitions)
Why did we want to calculate radar cross sections (RCSs) of spherical targets?
Everybody has a spherical RCS code, but …
Where did I start and what did I do?
There were some problems. Otherwise I would not have anything to talk about.
Algorithms were developed and a code was written. The code works -- some example calculations.
RCS of a Metal Sphere

\[ \frac{\sigma}{\pi a^2} \]

- Dipole Moment
- Front Face
- Creeping Wave
- Rayleigh Region
- Resonance Region
- Optics Region

\[ ka = \frac{2\pi a}{\lambda} \]

(sphere circumference in wavelengths)
Motivation for Having a Spherical RCS Code

- RCS of spherical targets needed for projects.
  - Balloons with metallic coatings or nose tips
    - Scattering possibly in resonant region

- Simple calculations to support several studies
  - Metallic coatings
    - Determine thickness needed to have RCS like solid metal
  - Effects of dielectric coatings
  - Lossy coatings
    - RCS reduction by dissipating creeping waves
Everybody has a spherical RCS code, but …

- Numerical overflow occurs when a layer consists of a real conducting or lossy material.
  - A lossy material is a material with a dielectric constant having a nonzero imaginary part.
  - Real conductors behave numerically like a very lossy material.
What did I do? (A little history)

♦ First, I tried a bigger computer
  • Cray permitted representing materials with larger losses but values were limited.
  • Overflow still occurred for real conductors.
♦ Then, I tried to modify Sandia codes
  • Impeded by lack of documentation.
♦ Finally, I decided to start from scratch
  • Used formulation found in Chapter 3 of Ruck, et. al., *Radar Cross Section Handbook*. 
The seas were rough -- there were problems

- There is an error in coefficients for RCS expression in Ruck handbook.
- Spherical Hankel functions are unbounded for complex arguments (the problem with most spherical RCS codes).
- The spherical Bessel and scaled spherical Hankel functions needed to be calculated.
- Computer arithmetic differs from real number arithmetic. (Not a show stopper, but a lesson learned.)
Calculating the RCS

♦ Recursive algorithm to obtain RCS.
  • Start with impedances and admittances at core layer
    – Expressions contain spherical Bessel and Hankel functions.
  • Then recursively generate impedances and admittances at successive layers until obtaining them at surface.
  • Calculate the Mie coefficients.
    – Used in an infinite series expression for the RCS.
    – Expressions use the impedances and admittances at surface and spherical Bessel and Hankel functions.
  • Calculate the RCS using the Mie coefficients.
The Problem with Spherical Hankel Functions and Its Solution

Unbounded as imaginary part decreases:

\[ h_n(z) = \exp(iz) \, h_n(z) \]

where \( i = \sqrt{-1} \)

\( h_n \) does not contain \( \exp() \) (the scaled spherical Hankel functions).

Solution:

- Calculate scaled spherical Hankel functions.
- Replace scaling factors in appropriate locations to get correct contributions for each layer.
Calculating the Scaled Spherical Bessel and Hankel Functions

♦ Use forward recursion when magnitude of argument is strictly greater than 0.2
  • Starting functions are trigonometric.
♦ Use backward recursion when magnitude of argument is less than or equal to 0.2.
♦ For large magnitude arguments, backward recursion does not work.
A Problem with Computer Arithmetic

♦ The problem

- Coding expressions as they appear in reference led to not getting RCS of 0 m² for a conducting shell with 0 thickness.
- Resulted from propagation of truncation errors.
  – Computer does not represent all real numbers exactly.

♦ A solution

- Careful ordering of the steps in coding the expressions to produce correct result.
The SPHERE Code

♦ Algorithms developed to calculate RCS of multilayered spherical targets
  • Recursive with layers
  • Scaled spherical Hankel functions
  • Forward or backward recursion to calculate values of spherical Bessel and Hankel functions
  • Careful ordering of steps to compute quantities needed.

♦ SPHERE code developed
Some Examples

♦ Metal sphere
♦ Metal sphere with thin dielectric coating
♦ Thin metallic shells.
♦ RCS reduction
RCS of 12-in Metal Sphere

![Graph showing the RCS of a 12-in Metal Sphere against Frequency (GHz). The y-axis represents RCS in dBsm, with values ranging from -10 to 0 in steps of 2. The x-axis represents Frequency in GHz, ranging from 0 to 8.]
RCS of Solid and Coated Spheres

Frequency (GHz)

RCS (dBsm)

Solid Metal
Coated k = (4,0)
RCS of Solid Sphere and Spherical Shells

![Graph showing RCS (dBsm) vs Frequency (GHz) for Solid Metal, 1000 A Shell, 100 A Shell, and 10 A Shell.]
RCS Reduction II

Range (m) -4 -3 -2 -1 0 1 2 3 4
RCS (dBsm) -90 -80 -70 -60 -50 -40 -30 -20 -10 0

Metal
Metal with (20,19) RAM

RCS (dBsm)
Range (m)
8-12 GHz

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