A new multi-scale, stabilized method for Q1/P0 finite element computations of Lagrangian shock hydrodynamics is presented. Instabilities (of hourglass type) are controlled by a stabilizing operator derived using the variational multi-scale analysis paradigm. The resulting stabilizing term takes the form of a pressure correction. With respect to currently implemented hourglass control approaches, the novelty of the method resides in its residual-based character. The stabilizing residual has a definite physical meaning, since it embeds a discrete form of the Clausius-Duhem inequality. Effectively, the proposed stabilization samples and acts to counter the production of entropy due to numerical instabilities. The proposed technique is applicable to materials with no shear strength, for which there exists a caloric equation of state. The stabilization operator is incorporated into a mid-point, predictor/multi-corrector time integration algorithm, which conserves mass, momentum and total energy. Encouraging numerical results in the context of compressible gas dynamics confirm the potential of the method.