Modeling Ice Sheets with MALI

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Brief introduction and motivation

- Greenland and Antarctica ice sheets store most of the fresh water on hearth.
- Modeling ice sheets (Greenland and Antarctica) dynamics is essential to provide estimates for sea level rise* and fresh water circulation.
- Global mean sea-level is rising at the rate of 3.2 mm/yr and the rate is increasing.
- Latest studies suggest possible increase of 0.3 – 2.5m by 2100.

*DOE SciDAC project **ProSPect** (Probabilistic Sea Level Projection from Ice Sheet and Earth System Models), Institutes: LANL, LBNL, SNL, ONL, NYU, Univ. of Michigan
Brief introduction and motivation

- Ice behaves like a very viscous shear-thinning fluid (similar to lava flow) driven by gravity. Source: snow packing/water freezing. Sink: ice melting / calving in ocean.

- Greenland and Antarctica have a shallow geometry (thickness up to 4 km, horizontal extensions of thousands of km).

Perito Moreno glacier

from http://www.climate.be
Outline:

- Ice sheet equations
- MALI model
- Model initialization
- Ensemble ice sheet modeling of ocean melt variability at Thwaites glacier
- Uncertainty Quantification: Inference and Forward propagation
Ice Sheet Modeling

Ice momentum equations

- **Ice flow equations** (momentum and mass balance)

\[
\begin{align*}
-\nabla \cdot \sigma &= \rho g \\
\nabla \cdot \mathbf{u} &= 0
\end{align*}
\]

Boundary condition at ice-bedrock interface:

\[(\sigma \mathbf{n} + \beta \mathbf{u})_\parallel = 0 \quad \text{on} \quad \Gamma_\beta, \quad \mathbf{u} \cdot \mathbf{n} = 0\]
Ice Sheet Modeling

Ice momentum equations

- Ice flow equations (momentum and mass balance)

\[
\begin{cases}
- \nabla \cdot \sigma = \rho g \\
\nabla \cdot \mathbf{u} = 0
\end{cases}
\]

Boundary condition at ice-bedrock interface:

\[
(\sigma \mathbf{n} + \beta \mathbf{u})_\parallel = 0 \quad \text{on} \quad \Gamma_\beta, \quad \mathbf{u} \cdot \mathbf{n} = 0
\]

with:

\[
\sigma = 2\mu \mathbf{D} - p\mathbf{I}, \quad D_{ij}(\mathbf{u}) = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

Nonlinear viscosity:

\[
\mu = \frac{1}{2} \alpha(T) |\mathbf{D}(\mathbf{u})|^{\frac{1}{n}-1}, \quad n \geq 1, \quad \text{(typically } n \approx 3)\]

Viscosity is singular when ice is not deforming

Stiffening/Damage factor

\[
\mu^*(x, y, z) = \phi(x, y) \mu(x, y, z)
\]

\(\phi\): stiffening factor that accounts for modeling errors in rheology
Ice Sheet Modeling

Main components of an ice model:

- **Ice flow equations** (momentum and mass balance)

\[
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\]

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- **Model for the evolution of the boundaries**
  (thickness evolution equation)

\[
\frac{\partial H}{\partial t} = H_{\text{flux}} - \nabla \cdot (H \bar{\mathbf{u}}), \quad \bar{\mathbf{u}} = \frac{1}{H} \int_z \mathbf{u} \, dz
\]

- Temperature, Basal hydrology
- Coupling with other climate components (e.g. ocean, atmosphere)
Stokes approximations in different regimes

\[ \begin{aligned}
-\nabla \cdot (2\mu D(u) - pI) &= \rho g \\
\nabla \cdot u &= 0
\end{aligned} \]

\[ D(u, v) = \left[\begin{array}{ccc}
 u_x & \frac{1}{2} (u_y + v_x) & \frac{1}{2} (u_z + w_x) \\
\frac{1}{2} (u_y + v_x) & v_y & \frac{1}{2} (v_z + w_y) \\
\frac{1}{2} (u_z + w_x) & \frac{1}{2} (v_z + w_y) & -(u_x + v_y)
\end{array}\right] \quad u := \left[\begin{array}{c}
 u \\
v \\
w
\end{array}\right]
\]

\[ \mu = \mu(|D(u, v)|) \]

Quasi-hydrostatic approximation

3rd momentum equation

\[ -\partial_x(\mu u_z) - \partial_y(\mu v_z) - \partial_z(2\mu w_z - p) = -\rho g, \]

continuity equation

\[ w_z = -(u_x + v_y) \]

\[ \Rightarrow p = \rho g(s - z) - 2\mu(u_x + v_y) \]

\[ -\nabla \cdot \left( 2\mu \tilde{D} - \rho g(s - z)I \right) = 0 \]

with \( \tilde{D}(u, v) = \left[\begin{array}{ccc}
 2u_x + v_y & \frac{1}{2} (u_y + v_x) & \frac{1}{2} u_z \\
\frac{1}{2} (u_y + v_x) & u_x + 2v_y & \frac{1}{2} v_z
\end{array}\right] \]

\footnote{Dukowicz, Price and Lipscomb, 2010. J. Glaciol}
Stokes approximations in different regimes

\[ \mathbf{D} = \begin{bmatrix} 0 & 0 & \frac{1}{2} u_z \\ 0 & 0 & \frac{1}{2} v_z \\ 0 & 0 & w_z \end{bmatrix} \quad \text{Ice regime:} \quad \text{grounded ice with frozen bed} \]

\[ p = \rho g (s - z) \]

\[ \mathbf{FO}(u, v) \]

\[ \mathbf{D} = \begin{bmatrix} u_x & \frac{1}{2} (u_y + v_x) & 0 \\ \frac{1}{2} (u_y + v_x) & v_y & 0 \\ 0 & 0 & w_z \end{bmatrix} \quad \text{Ice regime:} \quad \text{shelves or fast sliding grounded ice} \]

\[ p = \rho g (s - z) - 2\mu (u_x + v_y) \]

\[ \mathbf{SIA}(u, v) \]

\[ \mathbf{SSA}(u, v) \]

Shallow Ice Approximation

Shallow Shelf Approximation

Hybrid models, \( \approx \text{SIA} + \text{SSA} \)
MPAS-Albany Landice model (MALI)
Algorithm and Software

<table>
<thead>
<tr>
<th>ALGORITHM</th>
<th>SOFTWARE TOOLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite Volume on Voronoi Meshes</td>
<td>MPAS</td>
</tr>
<tr>
<td>Linear Finite Elements on test/hexas</td>
<td>Albany</td>
</tr>
<tr>
<td>Quasi-Newton optimization (L-BFGS)</td>
<td>ROL</td>
</tr>
<tr>
<td>Nonlinear solver (Newton method)</td>
<td>NOX</td>
</tr>
<tr>
<td>Krylov linear solvers/Prec</td>
<td>AztecOO/ML, Belos/MueLu</td>
</tr>
<tr>
<td>Automatic differentiation</td>
<td>Sacado</td>
</tr>
</tbody>
</table>

**MPAS**: Model for Prediction Across Scales, fortran finite volume library:
- works on Voronoi Tessellations
- conservative Lagrangian schemes for advecting tracers
- evolution of ice thickness

**Albany**: C++ finite element library built on Trilinos to enable multiple capabilities:
- Jacobian/adjoints assembled using automatic differentiation (Sacado).
- nonlinear and parameter continuation solvers (NOX/LOCA)
- large scale PDE constrained optimization (Piro/ROL)
- linear solver and preconditioners (Belos/AztecOO, ML/MueLu/Ifpack)

Hoffman, et al. GMD, 2018
Perego, Price, Stadler, JGR, 2014
MPAS-Albany Landice model (MALI)

Antarctic Ice Sheet velocity

Colored by ice sheet velocity surface velocity
(blue = slow, red = fast)
Model initialization
(using PDE-constrained optimization)

GOAL
Find ice sheet initial state that
- matches observations (e.g. surface velocity, temperature, etc.)
- is in compliance with flow model and climate forcing

estimating unknown (basal friction) or poorly known parameters (bed topography)

Bibliography
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- Price, Payne, Howat and Smith, PNAS, 2011
- Petra, Zhu, Stadler, Hughes, Ghantas, J. Glaciology, 2012
- Pollard DeConto, TCD, 2012
- W. J. J. Van Pelt et al., The Cryosphere, 2013
- Goldberg and Heimbach, The Cryosphere, 2013
- Michel et al., Computers & Geosciences, 2014
- Goldberg et al., The Cryosphere Discussions, 2015
**Deterministic Inversion**

PDE-constrained optimization problem: cost functional

**Problem**: find initial conditions such that the ice matches available observations.

**Optimization problem**:

find $\beta$ and $H$ that minimize the functional* $\mathcal{J}$

$$
\mathcal{J}(\beta, \phi) = \int_{\Omega} \frac{1}{\sigma_u^2} |u - u^{obs}|^2 ds 
+ \int_{\Omega} \frac{1}{\sigma_\phi^2} |\phi - 1|^2 ds 
+ \mathcal{R}(\beta, \phi)
$$

subject to ice sheet model equations

subject to ice sheet model equations (FO or Stokes)

$u$: computed depth averaged velocity

$\phi$: stiffening factor

$\beta$: basal sliding friction coefficient

$\mathcal{R}(\beta, \phi)$ regularization term

Greenland Inversion
velocity mismatch only, tuning basal friction

Inversion with 1.6M parameters

Estimated
Computed
Target

Basal friction coefficient (kPa yr/m)

surface velocity magnitude (m/yr)

Geometry (Morlighem et al., Nature Geo., 2014)
Antarctica Inversion
velocity and stiffening mismatches, tuning basal friction and stiffening

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**estimated basal friction coefficient [kPa yr/m]**

**estimated softness parameter [adim]**

**computed surface velocity [m/yr]**

**observed surface velocity [m/yr]**

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**simulation details**

- parameters: 2.5M
- unknowns: 30M
- machine: Edison (NERSC)
- cores: 8640
- nodes: 180
- hours: 18
Ice sheet response under extreme (unrealistic) forcing

ABUMIP targets the response of the ice sheet model to instantaneous removal of all ice shelves, to understand the sensitivity of ice sheet to extreme climate forcing.

Simulation by Tong Zhang and Matt Hoffman
Ensemble ice sheet modeling of ocean melt variability at Thwaites glacier
(slides and most of work courtesy of Matt Hoffman)
Ocean Water Masses Controlling Ice Melting

Dutrieux et al. 2014

AASW

CDW

Jenkins et al. 2016
Climate Variability affecting Antarctic subshelf melting

- El Nino/Southern Oscillation (2-7 yr)
- Southern Annular Mode (20-30 yr)
- Pacific Decadal Oscillation (15-25 yr, 50-70 yr)
- Atlantic Multidecadal Oscillation (50-80 yr)

How might climate variability affect marine ice sheet stability?
Model Setup

- MPAS-Albany Land Ice (MALI)
- 3d First-order momentum balance approx. (Blatter/Pattyn)
- Variable resolution regional grid (1-8 km)
- Thickness, bed elevation from BEDMAP2
- Linear basal friction law
- Basal friction parameter optimized from InSAR surface velocity
- Fixed temperature field (pers. comm. Frank Pattyn)
- Calving front fixed in time
- SMB from RACMO2

- Validated by observed grounding line flux transient
Results: single run (amplitude=300m, period =20yr)
Results: all ensembles

Mass loss delay enhanced by
- Larger amplitude
- Longer period

SLR is 10-40 mm less after 500 years

SLR is delayed 9-43 years
Mechanism for delay in mass loss

1. Asymmetric melt forcing – primary mechanism (~75% of delay)
Mechanism for delay in mass loss

2. Nonlinear ice dynamic response to ice shelf melting
   – secondary mechanism (~25% of delay)

- Decreasing melt → large decrease in mass loss
- Increasing melt → small increase in mass loss
Conclusion

Subshelf melt variability affects grounding line evolution and sea level contribution

• Variable runs always retreat less than steady runs
• Effects small (~3%) for realistic (?) modes of variability
• ~10% less SLR for plausible large amplitude, long period variability after hundreds of years
• Decadal rates of change can differ by up to 50%
  – implications for interpreting mass balance observations, e.g., GRACE

• Caveats: parameterized melt, simplistic variability, uncertain bed topography
Uncertainty Quantification

Ultimate goal: quantify the Sea Level Rise and related uncertainties

Current Work flow:

- Perform **adjoint-based deterministic inversion** to estimate initial ice sheet state (i.e. characterize the present state of ice sheet to be used for performing prediction runs).

- **Bayesian inference**: Gaussian posterior low-rank approximation; use deterministic inversion to characterize the parameter distribution (i.e, use the inverted field as mean field of the parameter distribution and approximate its covariance using sensitivities/Hessian).

- **Forward Propagation**

(sheperd et al. 2012)
Bayesian Inference
Gaussian approximation*

It is possible to approximate the distribution of the basal friction informed by the velocity data with a Gaussian distribution* using the Hessian of the objective functional of the initialization problem.

\[ \Sigma_{\text{post}} = \left( H + \Sigma_{\text{prior}}^{-1} \right)^{-1} = (\Sigma_{\text{prior}} H + I)^{-1} \Sigma_{\text{prior}} \]

Compute approximation of posterior using low-rank approximation:
\[ \Sigma_{\text{prior}} H \approx W_r \Lambda_r V_r^T \quad \text{(low rank approximation - randomized SVD)} \]

*T. Isaac, N. Petra, G. Stadler, O. Ghattas, JCP, 2015
Bayesian Inference
Gaussian approximation*

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\[ \Sigma_{\text{prior}} H \approx W_r \Lambda_r V_r^T \quad \text{(low rank approximation - randomized SVD)} \]

\[ \Sigma_{\text{post}} = \Sigma_{\text{prior}} - W_r \Lambda_r W_r^T + \mathcal{O} \left( \sum_{i=r+1}^{N} \frac{\lambda_i}{1 + \lambda_i} \right) \]

(Sherman-Morrison-Woodbury formula)

*T. Isaac, N. Petra, G. Stadler, O. Ghattas, JCP, 2015
Challenge: dimension of parameter space is too high for forward propagation. We want to mitigate this with a multifidelity approach, using lower fidelity models.
Bayesian Inference and Forward Propagation
dimension reduction by adding physics

Subglacial hydrology models rely on an handful of parameters that, to first approximation can be considered uniform.

Two-step estimation of basal friction parameters
Estimate spatial dependent basal friction by minimizing mismatch between observed surface velocity and FO surface velocity (usual basal friction estimation)
Calibrate the basal hydrology model by matching that (target) basal

Left: target basal friction [kPa yr/m], from FO calibration
Right: basal friction computed w/ calibrated hydrology model
Considerations on Bayesian Calibration and Uncertainty Propagation

- **Bayesian Inference:**
  High dimensional parameter space. Even if we accept the Gaussian approximation for the posterior, forward propagation is still unfeasible. Performing the Bayesian calibration to recover the true distribution for the parameters is also unfeasible.

- **Strategies for forward propagation:**
  - adopt a multifidelity strategy.
  - build emulator (polynomial chaos, Neural Networks, ...) of the forward model and sample emulator (issue: lots of model runs needed to build emulator)
  - use cheap physical models (e.g. SIA, SSA) or low resolution solves to reduce the cost of building the emulator.
  - use sensitivities and active subspace methods.
  - use techniques such as the compressed sensing to adaptively select significant modes and the basis for the parameter space.
  - Improve fidelity of the model (e.g. physical-based model for sliding considering subglacial hydrology) to reduce the parameter space.
Thank you!