

# Aggregate Rollover Coverage: A First-Best Solution for the Package-Shipping Industry

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# **Aggregate Rollover Coverage: A First-Best Solution for the Package-Shipping Industry**

## ABSTRACT

When specifying a new coverage option, underwriters must assess its viability with respect to profitability, consumer demand, and regulatory consent. This article specifies a new coverage option for clients of the package-shipping industry, who face transit risks of loss, damage, and delay. Currently, shipping companies and third-party insurers offer pay-as-you-go coverage options under per-claim deductible schemes, which tend to attract “low-volume” clients who suffer fluctuating losses. However, there are no standard coverage options for high-volume clients who find it cheaper to self-insure because their loss experience converges toward its asymptotic properties. In theory, aggregate deductibles offer an efficient alternative to self-insurance. In practice, however, aggregate deductible schemes are actuarially intractable because underwriters rarely know the precise number of shipments to be insured under a period of performance. This article introduces a “rollover” scheme that allows for aggregate deductibles, derives the conditions for a first-best solution, and describes the economic surplus created by the scheme.

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## **1. Introduction**

Package-shipping contracts specify a schedule of rates and fees, which are applied on a per-package basis according to the characteristics of the package itself (e.g. size, weight, contents, etc.) and the conditions for its shipment (e.g. distance, time-in-transit, etc.). Such schedules can also include fees to cover clients’ losses in the event of loss-of-package, physical damage to contents, or delay-in-shipment. Current standards for coverage in shipping contracts specify per-package coverage limits and per-package deductibles, but not aggregate deductibles. The findings of Eeckhoudt et al (1991) suggest that, under a scheme of constant per-package deductibles, the consumer demand for coverage decreases as the number of packages shipped by the client under the contract increases. Thus, high-volume clients will tend to self-insure. Li and Lui (2003) refer to

self-insurance as a third-best solution. They describe a second-best solution consisting of decreasing per-claim deductibles, which is applicable when each claim-payment can be calculated with respect to prior losses.

Arrow (1974) showed that a single aggregate deductible provides a first-best solution for contracts of multiple potential claims. Yet, aggregate deductible options are absent in the small-package shipping industry, where multiple claims are common for high-volume clients. This observation seems to suggest that there are practical disadvantages to aggregate deductibles.

Cohen (2002) suggests two disadvantages of aggregate deductibles to explain their “conspicuous” absence in automobile insurance policies. He first describes how verification (adjustment) costs can be higher under aggregate deductibles versus per-loss deductibles. He reasons that customers are less likely to file small claims under per-loss deductibles, whereas aggregate deductibles encourage customers to file all claims in order to increase the likelihood of achieving their aggregate deductible and thereby collecting any losses in excess of the deductible. Cohen predicates his analysis on the assumption that small losses are more likely than large ones, which we accept as true for both automobile and package coverage. His assumption supports his analysis of automobile insurance policies, which provide blanket coverage for both small and large losses that occur over the period-of-performance, but his assumption does not apply to package contracts. Specifically, under package contracts, the client is generally required to declare the value of each package, and premiums are assessed on each package based on its declared value. Under this scheme, any customer not intending to file a claim in the event of loss of a package has a negative incentive to purchase coverage on that package in the first place, so we would not expect a customer’s tendency to file a claim to be

lower under a per-package deductible than under an aggregate deductible. Therefore, aggregate deductibles will not lead to higher verifications costs in the case of package contracts.

Cohen identifies moral hazard as the second disadvantage to aggregate deductibles. He reasons that once a customer's losses exceed the aggregate deductible, the customer will have little or no incentive to take precautions to avoid losses. For package shipments, however, client play essentially no role in preventing a package from being lost or delayed, and only a minor role if any in preventing physical damage. In the case of physical damage, clients can often use more expensive packing materials and containers to reduce the likelihood of loss. However, coverage contracts can specify minimum standards for packing materials and containers as a condition of coverage, thereby preventing clients from downgrading to cheaper packing materials once cumulative claims exceed the aggregate deductible. Thus, the moral-hazard argument has little or no relevance in explaining the absence of aggregate deductible options in package contracts.

We submit that the primary difficulty prohibiting the use of aggregate deductibles in package contracts is actuarial in nature, and stems from the uncertainty regarding the number of package shipments to be covered under the period-of-performance. We overcome this actuarial difficulty by defining coverage in terms of "sequence-of-performance," which applies to a specified number of package shipments rather than a period of time. This modification allows the underwriter to derive a schedule of aggregate deductibles and corresponding lump-sum premiums. By introducing a rollover scheme, coverage can be reset from one sequence of packages to the next. The aggregate rollover coverage scheme requires only minor modifications to the underwriting and claims-adjustment processes that already exist for handling per-package deductibles.

## 2. The Model

We consider a risk of loss facing a sequence of package shipments:  $i = 1, 2, \dots, N$ . For simplicity, we assume that all packages have equal at-risk value  $v > 0$ . Each shipment is a Bernoulli trial facing risk of (full) loss with probability  $p > 0$ . The sequence of shipments results in a sequence of occurrences of loss:  $j = 1, 2, \dots, K$ , where  $0 \leq K \leq N$ . The cumulative loss realized from such a sequence is  $L \equiv vK \leq vN$ .

### *Risk and the Uninsured Firm*

The expected cumulative loss without insurance is

$$E[L] = \sum_{i=1}^N iv\theta_i, \quad (1)$$

where  $\theta_i \equiv \Pr[K = i | p, N]$ . We allow that the firm facing the risks described above is capable of estimating its expected loss. In the absence of an insurance option, the firm has the option of setting aside *loss capital* to cover planned loss  $\hat{L}$ , where  $(0 \leq \hat{L} \leq vN)$ . Loss capital is allocated at a constant cost-of-capital rate  $\phi \in (0, 1)$ , resulting in an expense for planned loss  $\phi\hat{L}$ . Let  $X \equiv X(\hat{L})$  denote *unplanned loss*, given by

$$X = \begin{cases} 0 & , \text{if } L \leq \hat{L} \\ L - \hat{L} & , \text{if } L > \hat{L} \end{cases} \quad (2)$$

We reasonably assume that firms are loss averse due to indirect costs associated with unplanned loss. Specifically, firms face increasing liquidity constraints when covering the replacement costs or consequential losses arising from unanticipated damage or loss of merchandise. Firms can also face increasing penalties as unplanned losses drive earnings below forecasts. Let  $C \equiv C(X | \hat{L})$  denote indirect cost as a function of unplanned loss.

Our assumption of increasing liquidity constraints and penalties implies  $C' > 0$ . Our analysis requires no assumptions regarding  $C''$ .

For a planned loss  $\hat{L}$ , let  $\hat{j} \equiv \hat{L}/v$  denote the threshold number of occurrences demarcating planned and unplanned loss. Since  $\theta_i$  denotes the probability of  $i$  occurrences from  $N$  packages, the expected indirect cost from excess loss is

$$E[C(X | \hat{L})] \equiv \sum_{i=\hat{j}+1}^N \left[ \theta_i \times \int_0^{(i-\hat{j})v} C(y) dy \right]. \quad (3)$$

The expected net final value of goods shipped without insurance is denoted by  $z_0$  and given by

$$z_0 = vN - E[L] - \phi\hat{L} - E[C(X | \hat{L})]. \quad (4)$$

The uninsured firm maximizes according to the first-order condition

$$\frac{\partial z_0}{\partial \hat{L}} = -\phi - \frac{\partial E[C(X | \hat{L})]}{\partial \hat{L}} \equiv 0, \text{ which implies}$$

$$\frac{\partial E[C(X | \hat{L})]}{\partial \hat{L}} = -\phi < 0. \quad (5)$$

That is, the uninsured firm increases its allocation of loss capital until the incremental reduction in expected indirect cost is less than the incremental cost of capital.

### ***An Insurance Scheme***

We now introduce an insurance policy to cover  $N$  package shipments subject to an aggregate deductible  $D$ . The policy is provided to the insured for a lump-sum premium of  $P$ , and expires after shipment of the  $N^{\text{th}}$  package. The sequences of shipments results in a sequence of claims  $j = 1, 2, \dots, K$ . Initially, each claim is applied to the deductible. Once the number of claims exceeds a threshold number of claims, given by  $d \equiv D/v$ , all

further claims are paid at full value  $v$ . That is, all claims  $j \leq d$  are applied to the deductible, and all claims  $j > d$  are paid in full.

Revising equation (1) to account for the deductible provides the expected direct loss under insurance:

$$E[L(D)] = \left[ \sum_{i=1}^d iv\theta_i \right] + D \sum_{i=d+1}^N \theta_i, \quad (6)$$

where  $\sum_{i=d+1}^N \theta_i = \Pr[K > d \mid p, N]$ . We also recognize that the insurance policy protects the

insured from the indirect cost of losses that are both unplanned and in excess of the deductible. Thus, unplanned loss from equation (2) is redefined as  $X \equiv X(\hat{L}, D)$  to account for the deductible, and is given by

$$X = \begin{cases} 0 & , \text{if } L \leq \hat{L} \\ L - \hat{L} & , \text{if } D > L > \hat{L} \\ D - \hat{L} & , \text{if } L > D > \hat{L} \end{cases} \quad (7)$$

It follows that the expected value of indirect costs under insurance is

$$E[C(X \mid \hat{L}, D)] \equiv \sum_{i=j+1}^d \left[ \theta_i \times \int_0^{(i-j)v} C(y) dy \right]. \quad (8)$$

The expected net final value of goods shipped under insurance is denoted by  $z(D)$  and given by

$$z(D) = vN - P - E[L(D)] - \phi\hat{L} - E[C(X \mid \hat{L}, D)]. \quad (9)$$

Let  $\Delta z \equiv z(D) - z_0$  denote the insured's net expected return from the insurance policy, given by

$$\Delta z = E[I(D)] - P + \sum_{i=\mu+1}^N \left[ \theta_i \times \int_0^{(i-\mu)v} C(y) dy \right], \quad (10)$$

where  $\mu = \max(\hat{j}, d)$  and  $E[I(D)] \equiv \sum_{i=d+1}^N (i-d)v\theta_i$  is the *expected* total indemnification to the insured under the policy.<sup>1</sup> The insurer's profit is premium minus indemnification and claims expense. Expected profit is

$$E[\pi] = P - E[I(D)] - E[G], \quad (11)$$

where  $E[G]$  is the expected claims expense.

### ***The Optimal Deductible for the Insured***

By solving equation (11) for premium and substituting into equation (10), the insured's net return can be expressed as

$$\Delta z = \sum_{i=\mu+1}^N \left[ \theta_i \times \int_0^{(i-\mu)v} C(y) dy \right] - E[\pi] - E[G]. \quad (12)$$

For simplification, we assume that the insurer's expected profit is a constant. Therefore, the insured's net return is a function of the deductible  $D$  and planned loss  $\hat{L}$ . The first-order conditions for maximization of the insured's net return are

$$\frac{\partial(\Delta z)}{\partial D} \equiv 0 \quad \text{and} \quad \frac{\partial(\Delta z)}{\partial \hat{L}} \equiv 0, \quad (13)$$

which can be rewritten as

$$\frac{\partial(\Delta z)}{\partial d} \equiv 0 \quad \text{and} \quad \frac{\partial(\Delta z)}{\partial \hat{j}} \equiv 0. \quad (14)$$

However, inspection of equation (12) reveals

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<sup>1</sup> The appendix provides a proof of equation (10).

$$\begin{aligned} \frac{\partial(\Delta z)}{\partial d} &= \frac{\partial}{\partial \mu} \left[ \sum_{i=\mu+1}^N \left[ \theta_i \times \int_0^{(i-\mu)v} C(y) dy \right] \right] \cdot \frac{\partial \mu}{\partial d} \quad \begin{cases} < 0 & \forall d \geq \hat{j} \\ = 0 & \forall d < \hat{j} \end{cases} \\ \frac{\partial(\Delta z)}{\partial \hat{j}} &= \frac{\partial}{\partial \mu} \left[ \sum_{i=\mu+1}^N \left[ \theta_i \times \int_0^{(i-\mu)v} C(y) dy \right] \right] \cdot \frac{\partial \mu}{\partial \hat{j}} \quad \begin{cases} < 0 & \forall d \leq \hat{j} \\ = 0 & \forall d > \hat{j} \end{cases} \end{aligned} \quad (15)$$

By principles of convergence, the first-order conditions in equation (14) are satisfied if and only if  $d = \hat{j}$ , which also means that  $D = \hat{L}$ . That is, the firm optimizes by allocating loss capital equal to the amount of the deductible. Since  $\hat{L} \equiv \hat{L}(p, \phi, C(X))$  is determined by satisfying equation (5), the firm's optimal aggregate deductible  $\hat{D}$  for the insurance policy is chosen accordingly:

$$\hat{D} = \hat{L}. \quad (16)$$

### ***Claims Expense and the Existence of a First-Best Insurance Solution***

Assuming the first-order conditions in equation (13) hold, a first-best premium  $P^*$  exists if there exists a premium for equations (10) and (11) that satisfies both  $\Delta z \geq 0$  and  $\pi \geq 0$ . Those equations and the result that  $d = \hat{j}$  provide a feasibility interval for  $P^*$ , given by

$$E[I(\hat{D})] + E[G] \leq P^* \leq E[I(\hat{D})] + \sum_{i=d+1}^N \left[ \theta_i \times \int_0^{(i-d)v} C(y) dy \right]. \quad (17)$$

To derive a more precise feasibility condition, we introduce a load factor  $\alpha$ , such that  $P = (1 + \alpha)E[I(D)]$  and therefore  $\alpha^* \Leftrightarrow P^*$ . Substitution into equation (17) yields a feasibility interval for the load factor, given by

$$\frac{E[G]}{E[I(\hat{D})]} \leq \alpha^* \leq \frac{\sum_{i=d+1}^N \left[ \theta_i \times \int_0^{(i-d)v} C(y) dy \right]}{E[I(\hat{D})]}. \quad (18)$$

Hence, a first-best premium exists if the following feasibility condition is satisfied:

$$0 \leq E[G] \leq \sum_{i=d+1}^N \left[ \theta_i \times \int_0^{(i-d)v} C(y) dy \right]. \quad (19)$$

In summary, an insurance solution exists for any prospective insured whose expected indirect cost of unplanned loss exceeds the insurer's expected claims expense.

To better understand the role of claims expense, we assume that claims expense is a constant function<sup>2</sup>  $E[G] = \bar{g} \cdot E[K]$ , where  $\bar{g}$  denotes the average expense per claim. We also note that  $E[I(\hat{D})] = v \cdot E[K]$ . Substitution into equation (18) yields

$$\frac{\bar{g}}{v} \leq \alpha^*, \quad (20)$$

where  $\frac{\bar{g}}{v}$  is the expense-to-indemnification ratio. Equation (20) provides several key results. First, it reveals that the expense-to-indemnification ratio provides the underwriter with a lower bound for the load factor. Second, it reveals the obvious relationship that cheaper claims operations will allow for lower load factors, making first-best coverage feasible to firms with lower loss aversion. Third, it reveals the subtler relationship that load factors can be lower for firms who ship higher-valued packages. Thus, cheaper claims operations will make first-best coverage feasible to firms who ship lower-valued packages.

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<sup>2</sup> We reasonably assume that claims associated with a single policy do not significantly impact the average cost in claims-adjustment operations, which typically service many insurance policies.

### ***Claims Expense and Market Surplus***

To clarify the benefits in an economic sense, we note that the market surplus provided by the coverage contract is defined by summing the insured's consumer surplus ( $\Delta z$ ) and the insurer's producer surplus ( $\pi$ ). From equations (10) and (11), the net combined consumer and producer surplus from enacting the policy is

$$surplus \equiv \pi + \Delta z = \sum_{i=d+1}^N \left[ \theta_i \times \int_0^{(i-d)y} C(y) dy \right] - E[G]. \quad (21)$$

Note that the market surplus will only be positive if the feasibility condition of equation (19) holds. So, equations (19) and (21) imply that a first-best premium exists and will create market surplus if claims-adjustment activity is cheaper than the indirect costs of unplanned losses.

### **3. Implementation: Why a Rollover Scheme?**

Successful implementation of aggregate deductibles by the underwriter relies on precise knowledge of the number of packages covered under the policy. Since the client's number of shipments typically fluctuates across any predefined time period, a period-of-performance contract will not allow for the necessary precision. What is required is a sequence-of-coverage endorsement, whereby each sequence-of-coverage insures a sequence of  $N$  packages with aggregate deductible  $D$ . The coverage provider keeps a running total of accumulated claims for any sequence of  $N$  packages, and pays all losses in excess of  $D$  for that sequence. Upon completion of one sequence, the balance of accumulated claims is reset to zero, and a new sequence begins. That is, the endorsement "rolls over" from one sequence of packages to the next, independent from the contract's performance period.

By defining a coverage contract in terms of estimable parameters, such as  $p, v, \bar{g}, E[L]$ , etc., the underwriter can devise a first-best premium  $P^*$ , if one exists, for any choice of deductible  $D$  and sequence size  $N$ :  $P = P(D, N | p, v, \bar{g}, E[L])$ . To assist brokers in marketing coverage options to prospective clients, the underwriter can provide a schedule of viable policies  $(P, D, N)$  for a range of  $D$  and  $N$ . If one or more solutions exist for a prospective client, then the client need only select the solution  $(P, \hat{D}, N)$  with the desired deductible  $\hat{D}$  and sequence size  $N$ . This approach would allow the client to readily determine which, if any, of a schedule of policy options is feasible and most desirable.

#### **4. Remarks**

Rollover of a sequence-of-performance endorsement can provide an easily implemented, economically efficient market solution for high-volume clients of the package-shipping industry. We have derived the conditions under which a solution exists for cases where the at-risk value per package is fixed and asymptotic properties are known to the underwriter.

The assumption that the at-risk value is equal across all packages can be easily extended for a broad class of clients who ship finite types of items, each with its own at-risk value. For example, consider a client who ships five different models of electronic equipment with five different at-risk values, and five different probabilities of occurrence of loss. In this case, a distinct sequence-of-coverage endorsement could be written for each model of equipment under one policy without actuarial difficulty. A more general extension to the current model would be required for cases in which at-risk values are not foreknown.

In cases where asymptotic properties are unknown or inadequately estimated, steps might be taken in implementation to facilitate the provision of rollover coverage. For example, if an underwriter is willing to select *a priori* estimates of asymptotic parameters to order to initiate coverage, then an endorsement might be written that allows parameters (such as the probability of occurrence of loss) to be updated and premiums to be adjusted with each rollover of the sequence-of-coverage. If *a priori* estimates are “close” to actual asymptotic properties, then clients will experience relatively small premium adjustments in the course of coverage and still be protected from potentially sizeable indirect costs of fluctuating losses.

## Appendix

PROOF OF EQUATION (10): Equations (4) and (9) imply  $\Delta z = E[L] - E[L(D)] + E[C(X | \hat{L})] - E[C(X | \hat{L}, D)] - P$ . By equations (1) and (6),  $E[L] - E[L(D)] =$

$$\left[ \sum_{i=d+1}^N iv\theta_i \right] + D \sum_{i=d+1}^N \theta_i = \sum_{i=d+1}^N (i-d)v\theta_i, \text{ which also equals the expected indemnification}$$

under the policy, denoted  $E[I(D)]$ . Let  $\Delta E[C] \equiv E[C(X | \hat{L})] - E[C(X | \hat{L}, D)]$ . By

$$\text{equations (3) and (8), } \Delta E[C] = \sum_{i=\hat{j}+1}^N \left[ \theta_i \times \int_0^{(i-\hat{j})v} C(y) dy \right] - \sum_{i=\hat{j}+1}^d \left[ \theta_i \times \int_0^{(i-\hat{j})v} C(y) dy \right]. \text{ In the case}$$

$$\text{of } \hat{j} \leq d, \Delta E[C] = \sum_{i=d+1}^N \left[ \theta_i \times \int_0^{(i-d)v} C(y) dy \right]. \text{ In the case of } \hat{j} \geq d, \sum_{i=\hat{j}+1}^d \left[ \theta_i \times \int_0^{(i-\hat{j})v} C(y) dy \right] = 0,$$

$$\text{which implies } \Delta E[C] = \sum_{i=\hat{j}+1}^N \left[ \theta_i \times \int_0^{(i-\hat{j})v} C(y) dy \right]. \text{ In both cases, } \Delta E[C] =$$

$$\sum_{i=\mu+1}^N \left[ \theta_i \times \int_0^{(i-\mu)v} C(y) dy \right] \text{ for } \mu = \max(\hat{j}, d). \text{ Q.E.D.}$$

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